Domestic and External Debt and Default[*]

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Abstract

We develop a general equilibrium model with defaultable domestic and external debt. Overlapping generations work, consume, accumulate capital and public debt. Successive, democratically elected governments choose taxes, public goods spending, domestic and external debt issuance and repayment. In politico-economic equilibrium, inter generational conflict strengthens debt capacity and lowers the fundamental conflict between creditor groups. Default decisions may or may not be correlated across debt tranches. Minimum debt returns raise the cost of public funds ex post and render default on other tranches more likely; ex ante, they crowd out capital. Under standard functional form assumptions the model is solved in closed form. Demographic ageing increases domestic debt capacity but has only modest effects on external debt capacity. Equilibrium default offers risk sharing possibilities, but only for select types of shocks.

1 Introduction

Public debt takes center stage in many economic crises past and present—including the ongoing European debt crisis. But it also serves as a useful policy instrument to economize on tax distortions, hedge, redistribute, or provide liquidity. An important body of economic literature studies these roles. Curiously, however, it is sharply divided into two strands, one analyzing domestic public debt and mostly focusing on optimal policy, and the other studying external public debt and emphasizing commitment problems and default.

This dichotomy is foreign to the public debate and often overly restrictive. After all, the main function of domestic and external public debt is identical: to smooth the shadow cost of public funds. Moreover, lack of commitment constrains the use of both types of debt, even in developed economies, and external and domestic debt crises often go hand

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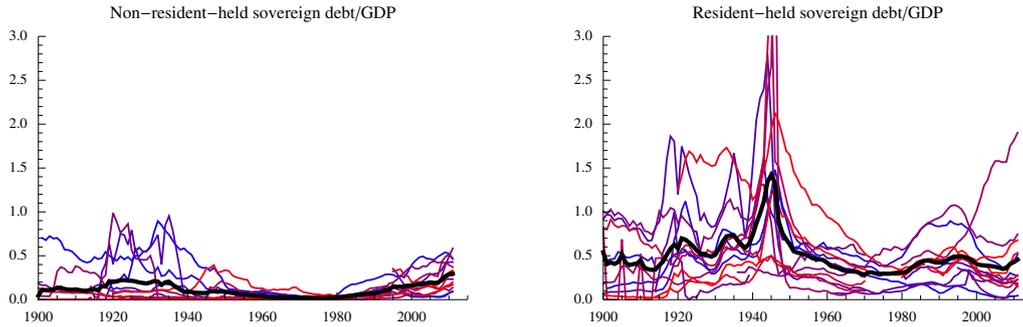
Figure 1: Non-resident- and resident-held sovereign debt, relative to GDP. Data source: Abbas et al. (2014). The black line indicates the unweighted average.

in hand (Reinhart and Rogoff, 2011). Finally, the ownership structure of public debt is an important policy instrument ex ante and state variable ex post, because a large domestic creditor base renders a run on public debt less likely, the cost of financing it lower, and a default decision easier to implement.[1] Accounting for and addressing such issues requires a unified theory of *defaultable* domestic *and* external debt. This paper lays out such a theory.

Reinhart and Rogoff (2008) document that domestic debt tends to exceed external debt. Figure 1, which uses Abbas, Blattner, De Broeck, El-Ganainy and Hu's (2014) data for 13 developed OECD economies over the last century, confirms this.[2] In fact, the dominance of domestically held debt is overwhelming once implicit liabilities are accounted for. Kaier and Müller (2015) estimate accrued-to-date implicit pension liabilities in eighteen member countries of the European Union to lie between 125 (Latvia) and 362 (France) percent of GDP in the year 2006.[3] Based on these estimates, resident-held debt relative to GDP more than quintupled over the last century, see figure 2.[4]

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[1] Along these lines, it is often argued that debt quotas in Italy and Japan are sustainable in spite of their elevated levels; and that further government deficits in Japan in combination with a shift of creditor base might push the economy to a "tipping point."

[2] Abbas et al.'s (2014) data set covers Australia, Belgium, Canada, France, Germany, Ireland, Italy, Japan, The Netherlands, Spain, Sweden, the United Kingdom, and the United States during the period 1900–2011 (debt-to-GDP ratio and share of non-resident-held debt). When data on non-resident-held debt is not available Abbas et al. (2014) assume that foreign investors hold all foreign currency debt but no domestic currency debt.

[3] Kaier and Müller's (2015) data set covers Austria, Belgium, Czech Republic, Germany, Spain, Finland, France, Greece, Hungary, Italy, Lithuania, Latvia, Malta, The Netherlands, Poland, Portugal, Sweden, and Slovakia. For broader measures of the "sustainability" of public finances in the European Union, see for example European Commission (2016). Member countries will soon have to publish estimates of unfunded public pension entitlements.

[4] Pension expenditures as a share of GDP and estimated implicit-debt-to-GDP ratios are tightly positively correlated in the cross section (Holzmann, Palacio and Zviniene, 2004; Kaier and Müller, 2015). This suggests that implicit pension liabilities have strongly risen over the last decades. To construct figure 2, we assume that all implicit debt was introduced after the year 1900 and that it increased uniformly. The right panel in figure 2 covers the countries in the intersection of Abbas et al.'s (2014) and Kaier and
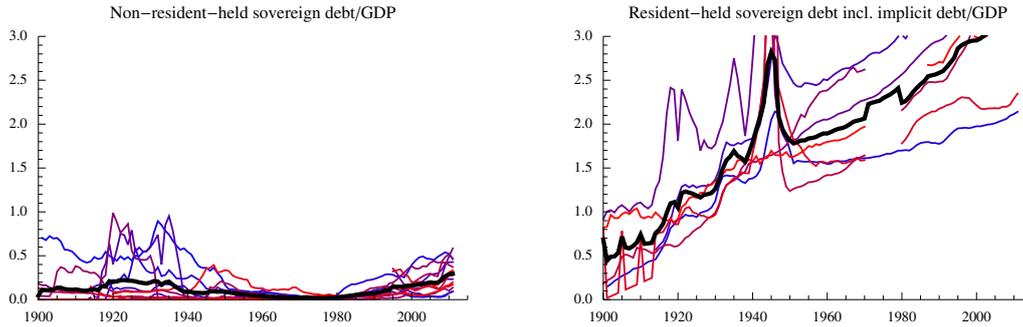
Figure 2: Non-resident- and resident-held sovereign debt incl. implicit debt, relative to GDP. Data sources: Abbas et al. (2014) and Kaier and Müller (2015). The black line indicates the unweighted average.

This paper proposes a unified theory of defaultable domestic and external debt. Its aim is to bridge the gap between the macroeconomic/public finance perspective on public debt on the one hand and the international finance perspective on the other, and to explain the observed unequal growth of domestic and external debt. We develop a model that endogenizes multiple private and public sector choices and accommodates various types of shocks, and we solve it in closed form. We find that the model qualitatively accounts for the secular trend depicted in figure 2 and derive several additional testable implications.[5]

The model features a domestic private sector, external lenders, and political decision makers that are democratically elected in each period. The domestic private sector is composed of overlapping generations. When young, households work, pay taxes, consume, and accumulate capital and public debt; when old, they redeem their assets, pay taxes, and consume. All households in a period vote. The elected government sequentially (Markov perfect equilibrium) chooses domestic and external public debt issuance,[6] debt repayment rates, public goods spending, and taxes. Default may be selective and partial.[7] Shocks to productivity, demographics and other fundamentals buffet the economy. Macroeconomic

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Müller's (2015) data sets.

[5]For instance regarding the correlation of default choices across debt tranches, the role of conflict for debt capacity and government hedging policies, or the political choice of debt guarantees. Due to the limited availability of cross-country and time-series debt statistics, only few of these predictions can be empirically assessed at this point.

[6]According to Reinhart and Rogoff (2008), "[d]omestic public debt is issued under home legal jurisdiction. In most countries, over most of their history, it has been denominated in the local currency and held mainly by residents. By the same token, the overwhelming majority of external public debt— debt under the legal jurisdiction of foreign governments—has been denominated in foreign currency and held by foreign residents." Our use of the terms "domestic" and "external" debt corresponds with these historical regularities.

[7]This is the central friction we are interested in which undermines Ramsey policies. We disregard other frictions, for example tax distortions as in Lucas and Stokey (1983) which could be introduced (see Gonzalez-Eiras and Niepelt, 2008).

outcomes in politico-economic equilibrium reflect these shocks as well as the domestic and external default risk.

Default benefits taxpayers because the government must transfer fewer funds to lenders, and it hurts creditors. In the case of domestic debt, the creditors are old, domestic households who invested in government bonds along with physical capital during their youth. In the case of external debt, the creditors are foreigners. Whether the government defaults on a debt tranche depends on how attractive political decision makers find it to benefit taxpayers at the cost of creditors. A sufficient statistic for the benefit of defaulting is the shadow cost of public funds—the social cost as perceived by political decision makers of replenishing government funds. This shadow cost is tightly connected to the cost of extracting resources from households, thus linking government finance and the domestic wealth distribution (see below).

A haircut on external debt triggers deadweight losses, either because renegotiation with external creditors is time consuming and disruptive or because the terms of the debt contract cannot be renegotiated at all and a default is socially costly. Possible sources of social costs include disruptions in the external or the financial sector; market exclusion; or loss of reputation.[8] We do not take a position on the specific mechanism through which such costs come about and instead opt for generality and tractability.

A default on domestic debt does not trigger deadweight losses because the terms of such debt can easily be renegotiated. The government's incentive to repay domestic liabilities therefore solely derives from the desire to avoid unwarranted redistribution between domestic bond holders and taxpayers. The assumption that domestic debt can be renegotiated at lower deadweight losses than external debt is not central for the model's results but very realistic (and it improves tractability). After all, reneging on the terms of government debt issued under domestic law and held by domestic creditors is relatively straightforward for lawmakers and much less costly than renegotiating the terms of external liabilities. The return on resident-held debt can easily be taxed; financial repression tactics can be employed to lower returns; and since domestic debt typically is denominated in local currency, a government may expropriate by fabricating surprise inflation and trying to disguise the induced debt devaluation to the general public.[9] Moreover, as documented above, large parts of domestic government debt take the form of implicit liabilities (in particular, unfunded social security promises) whose value can be—and often is—renegotiated and reset in the legislative process, for instance by changing taxes or wage and price indexation clauses.[10]

Debt service, public goods provision and the other policy choices reflect the resolution

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[8]See, for example, Reinhart and Rogoff (2004), Sturzenegger and Zettelmeyer (2006, pp. 49–52) or Panizza, Sturzenegger and Zettelmeyer (2009) for a discussion of the costs of sovereign defaults.

[9]We do not distinguish between outright and indirect forms of default. Even with the former, defaulting against domestic creditors is less costly than defaulting against foreigners. Reinhart and Rogoff (2008) document the frequency of outright and indirect defaults. Hall and Sargent (2011) document the role of inflation in lowering public debt/GDP ratios in post war US data.

[10]Soto, Clements and Eich (2011) report estimates according to which pension reforms in Eastern Europe led to changes in implicit government liabilities on the order of one to two GDPs. Kaier and Müller (2015) argue that already accrued pension rights typically are less affected by a reform; often, they are protected by law or even by the constitution.

of conflicts between interest groups. We assume that this resolution takes place in the context of a probabilistic voting game between political candidates.[11] In equilibrium, the government thus implements policies that maximize a weighted average of the welfare of all groups of voters and changes in the economy's demographic structure lead to gradual changes in policy outcomes.

An interior choice of domestic debt repayment rate lets the government equalize the cost-benefit ratio of taxation across cohorts. A binding floor on the domestic debt return undermines such cost-benefit smoothing; it raises the shadow value of public funds and reduces the return on external debt. In contrast, a binding floor on external debt service does not undermine efficient revenue collection. In this sense, restrictions on domestic burden sharing are particularly costly.

Even if cost-benefit ratios of taxation *are* equalized across cohorts this does not trivially correspond to an equalization of marginal utilities as one might expect—domestic burden sharing does *not* go hand in hand with domestic consumption smoothing. In fact, debt returns and thus, taxes tend to be higher than an equalization of consumption of the old and the young would suggest (controlling for differences in political influence across groups). Underlying this "overpayment" or "over taxation" result is the presence of general equilibrium net benefits of taxation which voters—unlike individual savers— internalize.

The general equilibrium net benefits arise because taxation depresses capital accumulation and thus, increases interest rates while lowering wages of subsequent cohorts. The political process internalizes the former effect because it concerns the current young who vote, but it does not internalize the latter effect which only affects future generations who do not vote. The political process also internalizes how current policy affects debt prices and future policy instruments of relevance for current voters by way of changing future state variables. The general equilibrium net benefits and price effects of taxation drive a wedge between the social marginal utility of young households and the shadow cost of public funds and thus, between the social marginal utilities of young and old households. The shadow cost of public funds is lower than in the absence of general equilibrium net benefits, and the return on domestic or external debt higher. In this sense, intergenerational conflict helps support the return on all debt tranches.

A return floor constitutes a form of partial commitment. Individual creditors are indifferent with respect to such a floor since they form rational expectations and always recover, on average, the required rate of return on their bond portfolio. For voters, in contrast, a return floor matters because they internalize the general equilibrium consequences associated with it. A higher floor raises the proceeds from the bond sale and it crowds out capital; this raises the interest rate; and the effect on state variables alters future policy and thus, affects the welfare of the current young once they have turned old.

Under standard functional form assumptions (mainly logarithmic utility and Cobb-Douglas production) we solve the model in closed form. We prove a series of comparative statics results. For example, we show that a return floor on domestic debt does *not* affect the share of output spent on external debt returns—an extreme case of "non conflict"

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[11]See Lindbeck and Weibull (1987).

between groups of bond holders—or on public goods spending. But of course, absent a Ponzi scheme on domestic bond holders, such a domestic return floor does increase tax rates. We also show that the return on external and domestic debt is proportional to domestic income although the cost of external default is independent of income. In the model, income dependence of debt returns directly reflects the counter-cyclicality of the shadow cost of public funds.

Maybe most importantly, we show that demographic ageing (a slowdown of the population growth rate) unambiguously increases domestic debt relative to GDP while the external debt-to-GDP-ratio may rise or fall, in line with observed long-term trends (see figure 2). Fundamentally, the rise of domestic debt is a consequence of the falling return on capital brought about by an ageing population. In response to the changed distribution of factor incomes the political process raises the debt service to the elderly. The model therefore links a fundamental trend observed over the last decades—the fall in real interest rates—to another one, the rise in domestic public indebtedness.

We also study a series of parametric examples and simulations. For example, we find that ageing leaves public goods purchases as a share of GDP nearly unchanged but leads to higher taxes, in line with the evidence; and that these effects are largely driven by the fact that voters internalize the general equilibrium net benefits of taxation and the effects of policy on debt prices.

In stochastic environments, we consider the effects of risky debt capacity and, taking up a recurrent theme in the macroeconomic/public finance literature, of the financing of "random wars." We find that default offers a means for the government to implement the politically desired wealth distribution, across generations or across groups of creditors, but it does not help to insure the government against financing needs associated with random variation in the need to purchase public goods and services. Intuitively, the state contingency of payoffs reflects the motive to implement the politically desired wealth distribution, but not the government's hedging needs.

Related Literature The macroeconomic/public finance literature on domestic public debt focuses on the crowding out of physical capital (Diamond, 1965); efficient tax smoothing (e.g., Barro, 1979; Lucas and Stokey, 1983; Angeletos, 2002; Aiyagari, Marcet, Sargent and Seppälä, 2002); and distributive tax shifting (e.g., Diamond, 1965; Niepelt, 2004). It also studies the role of public debt in providing liquidity (e.g., Woodford, 1990; Holmström and Tirole, 1998). These papers typically assume that the government can commit to servicing its debt even if it may lack the power to commit to other policy instruments.[12] While we also study some of these consequences of debt policy we adopt a more positive perspective, emphasizing conflicts of interest and international linkages, and assume that the government cannot commit.

The international finance literature studies the effects of commitment problems for external debt capacity and default (e.g., Eaton and Gersovitz, 1981; Eaton and Fernandez, 1995). It mostly abstracts from domestic conflict and debt, and it typically considers

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[12]For example, Lucas and Stokey (1983) posit that the government can commit to state contingent debt service but not to imposing ex-ante optimal tax rates.

endowment economies. In contrast, we incorporate defaultable domestic and external debt in a workhorse macroeconomic model with a neoclassical production function that employs capital and labor. In our model, general equilibrium effects crucially affect default and taxation decisions and domestic conflict is key. The general equilibrium effects arise *in spite of* an open capital account; they are present because foreigners cannot invest in domestic capital without being subjected to default risk. Also in contrast to the literature, we assume an overlapping generations structure. As a consequence, the model predicts international borrowing even if the discount factors of domestic households (and the government) and foreigners *coincide*, in stark contrast to representative agent sovereign debt models. Finally, we are able to exactly identify the different mechanisms by solving the model in closed form.

Aguiar and Amador (2011) develop a model of a small open economy where the incumbent government may expropriate capital in addition to loans extended by foreign investors and low external indebtedness fosters capital accumulation. In our model, in contrast, capital is owned by domestic households and cannot be expropriated; debt is held domestically and externally; and domestic conflict shapes default decisions. Moreover, we analyze the Markov perfect equilibrium while Aguiar and Amador (2011) focus on the best self-enforcing equilibrium.

Broner and Ventura (2016) analyze how external indebtedness may undermine a government's incentive to enforce private domestic debt service if it cannot discriminate against foreign lenders (see also Kremer and Mehta, 2000), for example because the presence of secondary markets renders it difficult to discriminate (Broner, Martin and Ventura, 2010). In contrast, we study public debt, abstract from secondary markets—not least because implicit government debt cannot be traded on secondary markets—, and allow for tranche specific repayment rates.

D'Erasmo and Mendoza (2016) consider an economy where households face idiosyncratic income shocks and use public debt to self insure, as in Aiyagari and McGrattan (1998), because there is no other asset. Di Casola and Sichlimiris (2015) and Azzimonti and Quadrini (2016) pursue related approaches. D'Erasmo and Mendoza (2016) assume that governments cannot selectively default against foreigners and that the default rate is either zero or unity. This generates a trade-off between the benefits of domestic debt, including self insurance, and the cost of external indebtedness. In our model, in contrast, households have a life-cycle savings motive; they save in government bonds and physical capital, and savings choices therefore affect production, linking our model to the macroeconomic literature; and the government's default choices are continuous and independent across debt tranches.[13]

Dovis, Golosov and Shourideh (2016) consider an environment without capital where taxes distort labor supply of households with unequal productivity. Lower taxes go hand in hand with higher government borrowing; they increase labor supply, which in turn gen-

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[13]D'Erasmo and Mendoza's (2016) analysis is motivated by the Spanish experience during the last decade while we are interested in longer-term developments, see figure 2. In order to being able to characterize equilibrium numerically, D'Erasmo and Mendoza (2016) assume that the weights in the government's social welfare function depend on households' income and bond holdings, and that exogenous default costs depend on the level of government consumption.

erates higher inequality. Since current labor supply is a bygone for future governments but not for the current one, the trade-off between external indebtedness and domestic inequality differs across successive governments. Dovis et al. (2016) show that this can lead to cyclical dynamics, where periods with high indebtedness and low inequality follow periods with low indebtedness and high inequality which they associate with "austerity." In our setting, households within a cohort are homogeneous and the key domestic conflict, concerning domestic debt service, plays out inter-generationally. Our model also incorporates capital which lies at the source of the general equilibrium effects that shape tax and default decisions.[14]

The choice of repayment rate on domestic debt in our model resembles the resolution of intergenerational conflict in Gonzalez-Eiras and Niepelt's (2008) model of social security.[15]

Outline The remainder of the paper is structured as follows. We layout the model in section 2 and define equilibrium in section 3. Section 4 contains a characterization of equilibrium and a first set of propositions. In section 5, we impose functional form assumptions. We characterize politico-economic equilibrium in closed form, establish additional results and present a series of numerical examples. Section 6 concludes.

2 The Model

2.1 Structure of the Economy

Time is discrete, indexed by t and extends over an infinite horizon. The economy is inhabited by two-period lived overlapping generations, competitive firms, a government that can be voted out of office in every period, and external lenders. The size of the young cohort in period t relative to the size of the old cohort is given by ν_t.

2.2 Households

Preferences of households born in period t are described by the utility function

$$u(c_t^y) + v_t(g_t) + \delta\mathbb{E}_t[u(c_{t+1}^o) + v_{t+1}(g_{t+1})], \qquad (1)$$

where $u(\cdot)$ and $v_t(\cdot)$ denote strictly increasing and concave utility functions; $\delta \in (0,1)$ the discount factor; $\mathbb{E}_t[\cdot]$ the expectation conditional on information available at time t; (c_t^y, c_{t+1}^o) cohort t's consumption path over the life cycle; and g_t government spending for public goods. The preference for public goods may vary over time.

Young households are born without assets and inelastically supply one unit of labor at wage w_t. Their labor income is taxed at rate τ_t. They consume their after-tax labor income or invest it in one of two assets: physical capital, k_{t+1}; or domestic government

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[14]Paczos and Shakhnov (2014) constitutes another recent attempt to integrate the analysis of domestic and external debt.

[15]For an analysis of the equivalence between social security and debt regimes with and without commitment, see Gonzalez-Eiras and Niepelt (2015).

debt, d_{t+1}. Capital has a price of unity and yields a state-contingent gross return R_{t+1} in the subsequent period. Debt trades at price q_t^d and pays a gross return r_{t+1}^d. Returns are taxed at the same rate as labor income. Consumption in young and old age, respectively, thus equals

$$
\begin{aligned}
c_t^y &= w_t(1 - \tau_t) - k_{t+1} - q_t^d d_{t+1}, & (2) \\
c_{t+1}^o &= (k_{t+1} R_{t+1} + d_{t+1} r_{t+1}^d)(1 - \tau_{t+1}). & (3)
\end{aligned}
$$

2.3 Firms

The aggregate production function is neoclassical and displays constant returns to scale in capital and labor. Capital depreciates fully after one period (corresponding to one generation). Output per old household equals $y_t = y_t(k_t, \nu_t)$. The function $y_t(\cdot)$ is indexed by time because parameters of the function may be exogenously time varying. Competitive factor markets imply

$$
\begin{aligned}
R_t &= \partial y_t(k_t, \nu_t)/\partial k_t, & (4) \\
w_t &= \partial y_t(k_t, \nu_t)/\partial \nu_t. & (5)
\end{aligned}
$$

2.4 External Lenders

External lenders are competitive. Their asset pricing kernel in period t, m_t, is exogenous from the perspective of the domestic economy. Lenders buy external debt e_{t+1} (per capita of young households) at price q_t^e from the government and receive a gross return r_{t+1}^e in the subsequent period.[16]

2.5 Government

The government finances public goods spending as well as the domestic and external debt service by levying taxes and issuing new debt. The budget constraint of the government reads (using constant returns to scale)

$$
g_t + d_t r_t^d + e_t r_t^e = \tau_t(y_t + d_t r_t^d) + \nu_t q_t^d d_{t+1} + \nu_t q_t^e e_{t+1}. \tag{6}
$$

The economy's resource constraint is given by

$$
g_t + c_t^o + \nu_t c_t^y + \nu_t k_{t+1} = y_t + \nu_t q_t^e e_{t+1} - e_t r_t^e. \tag{7}
$$

The return on debt is chosen ex post and restricted to be weakly positive: $r_t^d, r_t^e \geq 0$. A choice of low repayment rate on domestic debt harms domestic creditors, i.e. the old, and

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[16]External lenders do not buy debt from domestic households. This assumption is without loss of generality when the government can intervene in the capital account. If the "sovereign ceiling" imposed by the government effectively hands control over all external borrowing to the government, private external debt can be replicated by a government tax and debt policy. There exists ample evidence of private sector external debt that was absorbed by the public sector. A recent example are bank liabilities in Euro area countries that were guaranteed or eventually absorbed by national governments during the Euro area debt crisis. See also Reinhart and Rogoff (2011, p. 1692) on the Latin American debt crisis in the 1980s.

therefore may be politically costly (see below). Default on external debt is costly as well. We assume that the government's valuation of debt repayment to external lenders is given by $\varphi_t(e_t r_t^e)$ where $\varphi_t(\cdot)$ is weakly increasing and continuously differentiable. Function $\varphi_t(\cdot)$ can represent a welfare function—the government (or domestic lobby groups) value foreign lenders' welfare because it is more difficult to exploit future gains from trade if lenders are disgruntled; or it can represent a cost function that reflects the many default or renegotiation costs discussed in the sovereign debt literature.

We also allow for the possibility of *partial commitment*, reflected in a lower bound on debt returns, $\bar{r}_t^d$ or $\bar{r}_t^e$, chosen in the preceding period.[17]

2.6 Politics

The government's choice of policy instruments reflects societal preferences as aggregated in the political process. In each period, two candidates compete for office in democratic elections. A candidate represents a policy platform that specifies values for the policy instruments in question as well as an ideological position that is orthogonal to the policy platform.[18] The valuation of the ideological position differs across voters (even if they agree about the preferred policy platform) and is subject to random aggregate shocks which are realized after candidates have proposed their platforms.

In the resulting "probabilistic-voting" Nash equilibrium, the candidates maximize their respective vote shares by both proposing the same policy platform. This equilibrium platform maximizes a convex combination of the welfare of all voters (not only of the majority group) where the weights reflect group size and sensitivity of voting behavior to policy changes.[19] Groups that care more strongly about the policy platform relative to ideology have more political influence since they are more likely to alter their support in response to small changes in the proposed platform. In equilibrium, these "swing voters" thus tilt policy in their own favor. If all voters are equally responsive to changes in the policy platform, electoral competition implements the utilitarian optimum with respect to voters.

Formally, the political process in period t and thus, the government in power in that period maximizes the objective function

$$\omega_t \left(u(c_t^o) + v_t(g_t) \right) + \nu_t (u(c_t^y) + v_t(g_t) + \delta \mathbb{E}_t[u(c_{t+1}^o) + v_{t+1}(g_{t+1})]) + \varphi_t(e_t r_t^e). \qquad (8)$$

The per-capita political weight of old relative to young households, $\omega_t > 0$, reflects whether old voters respond more or less strongly to changes in the policy platform than young voters.[20]

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[17]A state contingent fixed cost of defaulting can be represented by $\bar{r}_t^e > 0$ and $\arg\max \varphi_t(e_t r_t^e)$ fluctuating between values above and below $\bar{r}_t^e > 0$. Empirical evidence supports the notion of temporary rather than permanent default costs, in particular in a model where one period corresponds to several years. Gelos, Sahay and Sandleris (2011) report that defaults, if resolved quickly, do not preclude sovereigns from re-accessing financial markets.

[18]The ideological position is permanent and cannot be credibly altered in the course of electoral competition.

[19]See Lindbeck and Weibull (1987) and Persson and Tabellini (2000) for discussions of probabilistic voting.

[20]We assume that only the government currently in power values external debt repayment in the period.

2.7 Timing

Events unfold as follows: At the beginning of period t the exogenous state, including the parameters of the production and default cost functions as well as the valuation of public goods spending, is realized and becomes commonly known,

$$\hat{z}_t \equiv (\nu_t, v_t(\cdot), y_t(\cdot), \varphi_t(\cdot), \omega_t, m_t).$$

Next, a candidate is elected for office. When choosing among the competing candidates, voters take both direct and indirect policy implications for economic outcomes into account. The winning candidate implements her policy platform

$$\pi_t \equiv (g_t, r_t^d, r_t^e, \tau_t, \tilde{d}_{t+1}, \tilde{e}_{t+1}, \bar{r}_{t+1}^d, \bar{r}_{t+1}^e)$$

where $\tilde{d}_{t+1}$ and $\tilde{e}_{t+1}$ denote the supply of debt for the domestic and external market, respectively. Finally, households and external lenders form expectations about future policy choices and a competitive equilibrium results.

The exogenous state $\hat{z}_t$ lies in a finite state space $\hat{Z}$ and follows a Markov process. The complete state in period t is given by

$$z_t \equiv (\hat{z}_t, k_t, \bar{r}_t^d, \bar{r}_t^e, d_t, e_t).$$

We restrict equilibrium policy functions to be functions of the contemporaneous state.[21]

3 Equilibrium

We start by defining the equilibrium in the private sector before turning to politico-economic equilibrium.

3.1 Private Sector Equilibrium

Young households choose consumption and their portfolio holdings in order to maximize (1) subject to their budget constraints, (2) and (3); prices and returns on capital, $(w_t, q_t^d, \{R_{t+1}\})$; as well as policy, $(\tau_t, \{r_{t+1}^d\})$. Optimal choices satisfy the first-order conditions[22]

$$u'(c_t^y) = \delta \mathbb{E}_t[u'(c_{t+1}^o) R_{t+1}(1 - \tau_{t+1})], \tag{9}$$

$$u'(c_t^y) = \delta \mathbb{E}_t\left[u'(c_{t+1}^o) \frac{r_{t+1}^d}{q_t^d}(1 - \tau_{t+1})\right]. \tag{10}$$

Alternatively, one could assume that $\varphi_t(\cdot)$ also enters the objective function of the government in period $t-1$ or even the objective functions of all governments prior to period t. Our approach is the simplest. Since $\varphi_t(\cdot)$ is a reduced-form specification, choosing the simplest specification appears adequate.

[21]For an exposition of Markov perfect equilibrium, see Krusell, Quadrini and Ríos-Rull (1997).

[22]We will adopt assumptions under which private sector equilibrium choices are interior.

The consumption of old households is determined by the budget constraint, (3); their asset holdings, (k_t, d_t); the return on capital, R_t; and policy, (τ_t, r_t^d). Factor prices are determined according to (4) and (5).

Conditional on $\hat{z}_t$ and $\{\pi_{t+1}\}$, external lenders price the debt to guarantee the required rate of return,

$$q_t^e = \mathbb{E}_t[m_t r_{t+1}^e]. \tag{11}$$

Definition 1. A *private sector equilibrium* in period t conditional on the state, z_t; expected subsequent exogenous states, $\{\hat{z}_{t+1}\}$; as well as current and expected policy, $(\pi_t, \{\pi_{t+1}\})$, is a set of

    i. factor prices, $(w_t, R_t, \{R_{t+1}\})$;

    ii. debt prices, (q_t^d, q_t^e);

    iii. consumption levels, $(c_t^y, c_t^o, \{c_{t+1}^o\})$; and

    iv. domestic and external portfolio choices, (k_{t+1}, d_{t+1}) and e_{t+1},

such that the following conditions are satisfied:

    i. (2), (3), (4), (5), (9), (10) in period t; and (3), (4) in period $t + 1$;

    ii. the markets for domestic and external debt clear at prices (q_t^d, q_t^e), $d_{t+1} = \tilde{d}_{t+1}$ and $e_{t+1} = \tilde{e}_{t+1}$; and

    iii. condition (11) is satisfied.

For future reference, note that combining the optimality conditions of a young household yields

$$u'(c_t^y)(k_{t+1} + q_t^d d_{t+1}) = \delta \mathbb{E}_t[u'(c_{t+1}^o)(k_{t+1} R_{t+1} + d_{t+1} r_{t+1}^d)(1 - \tau_{t+1})]. \tag{12}$$

3.2 Politico-Economic Equilibrium

Let $\pi(\cdot) \equiv (g(\cdot), r^d(\cdot), r^e(\cdot), \tau(\cdot), \tilde{d}'(\cdot), \tilde{e}'(\cdot), \bar{r}^{d\prime}(\cdot), \bar{r}^{e\prime}(\cdot))$ denote policy functions (of the state) that are expected to govern other governments' policy choices.

The government in period t chooses π_t in order to maximize (8) subject to four constraints. First, that $\pi_{t+1} = \pi(z_{t+1})$ for every possible realization of z_{t+1}. Second, that the government budget constraint, (6), is satisfied. Third, that return floors (if they have been imposed) and non-negativity constraints are respected: $r_t^d \geq \bar{r}_t^d \geq 0$ and $r_t^e \geq \bar{r}_t^e \geq 0$. And fourth, that current and future policy, $(\pi_t, \{\pi_{t+1}\})$, together with $z_t, \{\hat{z}_{t+1}\}$ support a private sector equilibrium in period t which in turn may affect z_{t+1}.

Definition 2. A *politico-economic equilibrium* in period t conditional on the state, z_t; expected subsequent exogenous states, $\{\hat{z}_{t+1}\}$; and policy functions, $\pi(\cdot)$, is a set of

    i. policy choices, π_t; and

ii. a private sector equilibrium in period t conditional on z_t, $\{\hat{z}_{t+1}\}$, and $(\pi_t, \{\pi_{t+1}\})$,

such that the following conditions are satisfied:

   i. the policy choice, π_t, maximizes the objective function (8);

   ii. $r_t^d \geq \bar{r}_t^d \geq 0$ and $r_t^e \geq \bar{r}_t^e \geq 0$;

   iii. the government budget constraint, (6), is satisfied; and

   iv. $\{\pi_{t+1}\}$ is consistent with $\pi(\cdot)$ and the law of motion of the state as determined by the private sector equilibrium and the law of motion for the exogenous state.

In a rational expectations equilibrium, the anticipated policy functions are consistent with the actual policy decisions that a government in power takes. Moreover, the sequence of period-t politico-economic equilibria is consistent with the law of motion for the state.

Definition 3. A *politico-economic equilibrium* conditional on the initial state, z_0; and processes for the exogenous state, $\{\{\hat{z}_{t+1}\}\}_{t \geq 0}$, is a set of

   i. policy functions, $\pi(\cdot)$;

   ii. a sequence of the state, $\{z_t\}_{t>0}$; and

   iii. politico-economic equilibria in periods $t = 0, 1, 2, \ldots$ conditional on z_t, $\{\hat{z}_{t+1}\}$ and $\pi(\cdot)$,

such that the following conditions are satisfied:

   i. in each period t and each state the equilibrium choice of policy, π_t, corresponds with the policy functions, $\pi(\cdot)$, evaluated at the state; and

   ii. $\{z_t\}_{t>0}$ is consistent with the law of motion of the state, as determined by policy, the private sector equilibrium and the law of motion for the exogenous state.

We now characterize equilibrium.

4   Characterization of Equilibrium

4.1   Government Program

Since the objective functions and constraints of all agents including the government only involve the products of debt quantities and their returns or prices, debt quantities are not determined in equilibrium: the effects of an increase of debt quantity can be undone by a corresponding decrease of the debt price, return and return commitment.[23] Without loss of generality, the policy functions $\tilde{d}'(\cdot)$ and $\tilde{e}'(\cdot)$ as well as the equilibrium debt levels d_{t+1} and e_{t+1} therefore can be normalized to unity in all periods and histories.

---

[23]This hinges on the fact that old households are homogeneous such that the distribution of debt holdings in the population is trivial. See Gonzalez-Eiras and Niepelt (2015) for a discussion and analysis.

Proposition 1. The policy functions $\tilde{d}'(\cdot)$ and $\tilde{e}'(\cdot)$ satisfy $\tilde{d}'(\cdot) = \tilde{e}'(\cdot) = 1$; and $d_{t+1} = e_{t+1} = 1$ in all periods and histories. Accordingly, we have

$$\pi_t = (g_t, r_t^d, r_t^e, \tau_t, \bar{r}_{t+1}^d, \bar{r}_{t+1}^e), \ z_t = (\hat{z}_t, k_t, \bar{r}_t^d, \bar{r}_t^e).$$

From definition 2, the (remaining) policy choices in a politico-economic equilibrium in period t solve the following program, conditional on z_t, $\{\hat{z}_{t+1}\}$ and $\pi(\cdot)$:

$$
\begin{aligned}
\max \quad & \omega_t \left(u(c_t^o) + v_t(g_t) \right) + \nu_t(u(c_t^y) + v_t(g_t) + \delta \mathbb{E}_t[u(c_{t+1}^o) + v_{t+1}(g(z_{t+1}))]) + \varphi_t(r_t^e) \\
\text{s.t.} \quad & g_t + r_t^d + r_t^e = \tau_t(y_t + r_t^d) + \nu_t q_t^d + \nu_t \mathbb{E}_t[m_t r^e(z_{t+1})], \\
& c_t^o = (k_t R_t + r_t^d)(1 - \tau_t), \\
& c_t^y = w_t(1 - \tau_t) - k_{t+1} - q_t^d, \\
& k_{t+1} = w_t(1 - \tau_t)\kappa_t^1(\zeta_t), \\
& q_t^d = w_t(1 - \tau_t)\kappa_t^2(\zeta_t), \\
& c_{t+1}^o = \left(k_{t+1} R_{t+1}(k_{t+1}) + q_t^d \frac{r^d(z_{t+1})}{q_t^d} \right) (1 - \tau(z_{t+1})), \\
& r_t^d \geq \bar{r}_t^d, \ \ r_t^e \geq \bar{r}_t^e.
\end{aligned}
$$

The first constraint reflects the government budget constraint and rational expectations by foreign lenders, conditions (6) and (11) respectively. The second constraint is the budget constraint of old households in period t, (3), and the third constraint the budget constraint of young households, (2). (Recall that debt quantities are normalized to unity.)

Constraints four and five constitute fixed point conditions that pin down young households' asset purchases. The conditions encapsulate the budget constraints, (2) and (3); the equilibrium condition for the interest rate, (4); as well as the Euler equations (9)–(10). The κ_ts denote the shares of workers' disposable income invested in capital and domestic debt, respectively. These shares are functions of $\zeta_t \equiv (w_t(1 - \tau_t), \{\bar{r}_{t+1}^d, \bar{r}_{t+1}^e\}, \{\hat{z}_{t+1}\}, \pi(\cdot))$ that is, of all factors that are exogenous to the portfolio choice problem of young households: disposable income as well as future net asset returns as determined by the equilibrium policy functions, the exogenous state and $(\bar{r}_{t+1}^d, \bar{r}_{t+1}^e)$. The shares do not depend on k_{t+1} (although R_{t+1}, r_{t+1}^d and τ_{t+1}—which affect households' portfolio choices—do depend on k_{t+1}), due to the fixed point.

The sixth constraint combines the budget constraint of old households in period $t+1$, (3); the equilibrium condition for the interest rate, (4); and the policy functions for r_{t+1}^d and τ_{t+1}. Note that the right-hand side of the condition is expressed in a way that clearly distinguishes between the quantities of assets and their rates of return. This distinction will be important when we assess the welfare effects of policy induced changes in asset purchases and returns. From the usual envelope condition argument (or the Euler equation), the welfare effect of a marginal increase of asset purchases is of second order. The *indirect* welfare effects of induced changes in asset returns, however, are of first order.

Finally, the last constraint reflects the return floors.

We now turn to an analysis of the government's policy choices. As is evident from the government's program these choices affect the allocation both directly and indirectly,

through induced future policy changes. The latter channel is present either if the government chooses a component of z_{t+1} (that is, $\bar{r}^d_{t+1}$ or $\bar{r}^e_{t+1}$) or if it chooses a component of ζ_t which affects $k_{t+1} \in z_{t+1}$ (that is, τ_t, $\bar{r}^d_{t+1}$ or $\bar{r}^e_{t+1}$).

4.2   Public Goods Spending

Let μ_t denote the shadow cost of the government budget constraint in the program of the government in period t. The first-order condition with respect to g_t,

$$(\omega_t + \nu_t)v'_t(g_t) = \mu_t, \tag{13}$$

states that the social benefit (as perceived in the political process) of higher public goods spending and its budgetary cost are equalized. If government resources are scarce (μ_t is high) then public goods spending is low; this harms households.

4.3   Return on Domestic and External Debt

The equilibrium conditions for r^d_t are given by

$$\omega_t u'(c^o_t) - \mu_t \leq 0, \quad r^d_t \geq \bar{r}^d_t \tag{14}$$

as well as the complementary slackness condition. Raising the repayment rate on domestic debt benefits the old, who hold it, but imposes a burden on the government budget. In equilibrium, marginal costs and benefits are equalized unless the inequality constraint binds.

From (13), a binding inequality constraint implies that $\omega_t u'(c^o_t) < (\omega_t + \nu_t)v'(g_t)$—a waste of resources. Ceteris paribus, public goods spending then is lower and the shadow cost of public funds higher than in the absence of a binding return floor. To see this, suppose that the binding constraint $r^d_t \geq \bar{r}^d_t$ is relaxed. From (14) and holding τ_t fixed, the equilibrium values of r^d_t and/or μ_t then must fall. If only r^d_t fell and μ_t remained unchanged then equilibrium g_t and r^e_t would remain unchanged as well and the budget constraint would be violated (from (13), (15), and (6)). If instead r^d_t remained unchanged and μ_t fell then equilibrium g_t and r^e_t (weakly) would increase and the budget constraint would again be violated. We conclude that both r^d_t and μ_t must fall; accordingly, g_t must rise.

The equilibrium conditions for r^e_t are given by

$$\varphi'_t(r^e_t) - \mu_t \leq 0, \quad r^e_t \geq \bar{r}^e_t \tag{15}$$

as well as the complementary slackness condition. They reflect the same type of trade-off as the one governing the choice of r^d_t.

Proposition 2. If the inequality constraints $r^d_t \geq \bar{r}^d_t$ or $r^e_t \geq \bar{r}^e_t$ bind, then the shadow cost of public funds exceeds the social marginal utility of consumption of the old or the social marginal utility of external debt service, respectively. In either case, public goods spending is lower and the shadow cost of public funds higher, conditional on τ_t, than if the constraint(s) were relaxed.

4.4 Taxes

The choice of tax rate differs from the previously discussed government decisions in so far as it affects future state variables. The first-order condition with respect to τ_t reads

$$\omega_t u'(c_t^o)(k_t R_t + r_t^d) + \nu_t \left(u'(c_t^y)w_t - \mathcal{B}_t^\tau\right) = \mu_t \left(y_t + r_t^d + \nu_t \mathcal{Q}_t^\tau\right) \tag{16}$$

where $\mathcal{Q}_t^\tau \equiv \mathrm{d}(q_t^d + q_t^e)/\mathrm{d}\tau_t$ and $q_t^e = \mathbb{E}_t[m_t r^e(z_{t+1})]$. The first two terms on the left-hand side (featuring ω_t) represent the welfare losses from reduced consumption of old households, due to lower disposable income, and the third term represents the corresponding welfare loss of young households.

Lower disposable income also affects the savings choices of young households and thus, k_{t+1} and q_t^d. As discussed previously, this has second-order direct welfare consequences but first-order indirect consequences, by changing rates of return and policy in the next period. These indirect (general equilibrium and political) net benefits of higher taxes for young households are represented by the fourth term on the left-hand side, where

$$
\begin{aligned}
\mathcal{B}_t^\tau \equiv\ & \delta \mathbb{E}_t \left[u'(c_{t+1}^o) \left(k_{t+1}\frac{\mathrm{d}R_{t+1}(k_{t+1})}{\mathrm{d}\tau_t} + q_t^d \frac{\mathrm{d}(r^d(z_{t+1})/q_t^d)}{\mathrm{d}\tau_t} \right)(1 - \tau(z_{t+1})) \right. \\
& + u'(c_{t+1}^o)\left(k_{t+1}R_{t+1}(k_{t+1}) + r^d(z_{t+1})\right) \frac{\mathrm{d}(1 - \tau(z_{t+1}))}{\mathrm{d}\tau_t} \\
& \left. + v'_{t+1}(g(z_{t+1}))\frac{\mathrm{d}g(z_{t+1})}{\mathrm{d}\tau_t} \right].
\end{aligned}
$$

As savers, households take rates of return and future policy choices as given. As voters, in contrast, they are aware of their collective monopoly power vis-a-vis the subsequent cohort and government. They internalize that lower capital accumulation drives up interest rates (benefitting them) but may also lead to lower public goods spending or higher taxes (harming them).[24]

The terms on the right-hand side of first-order condition (16) represent the budgetary implications of a tax hike. There are two channels: one working through higher income taxes, where the tax base equals $y_t + r_t^d$; and the second through changed purchases of domestic and external debt, with $\nu_t \mathcal{Q}_t^\tau$ representing the change in debt sales.

The shadow cost of public funds in equation (16) represents a cost-benefit ratio—the marginal social cost of taxation relative to the marginal revenue gain for the government. It is useful to decompose (16) using two group specific cost-benefit ratios, one for the old and one for the "non-old" (the young and the external lenders). The former is given by

$$\mathrm{cb}_t^o \equiv \frac{\omega_t u'(c_t^o)(k_t R_t + r_t^d)}{k_t R_t + r_t^d}$$

and the latter equals

$$\mathrm{cb}_t^y \equiv \frac{\nu_t \left(u'(c_t^y)w_t - \mathcal{B}_t^\tau\right)}{\nu_t w_t + \nu_t \mathcal{Q}_t^\tau}.$$

---

[24]See the related discussion in Gonzalez-Eiras and Niepelt (2008). A smaller capital stock in the next period may also affect the choice of $\bar{r}_{t+2}^d$ and $\bar{r}_{t+2}^e$ but this does not affect welfare of households voting in period t.

Using these definitions, the shadow cost of public funds in equation (16) can be re-expressed as a weighted average of the group specific cost-benefit ratios. Letting ψ_t denote the olds' share of the revenue base, $\psi_t \equiv (k_t R_t + r_t^d)/(y_t + r_t^d + \nu_t \mathcal{Q}_t^\tau)$, we have

$$\mu_t = \psi_t \, \mathrm{cb}_t^o + (1 - \psi_t) \, \mathrm{cb}_t^y. \tag{17}$$

If the government had access to group-specific tax instruments then it could individually target each of the two cost-benefit ratios. It would optimally set them equal to each other and to the shadow cost of public funds. An unrestricted choice of domestic debt service in combination with the choice of tax rate allows the government to achieve the same result since taxes constitute a non-distorting instrument (at the time they are imposed) and domestic debt repayment transfers resources from the government to old households.

Formally, the shadow cost of public funds equals the social marginal utility of the old if r_t^d is unconstrained (proposition 2). Moreover, the social marginal utility of the old is identical to cb_t^o because all wealth and income of the old is taxed at rate τ_t. As a consequence, the optimality condition (17) reduces to

$$\mu_t = \mathrm{cb}_t^o = \mathrm{cb}_t^y$$

when r_t^d is unconstrained. Otherwise, μ_t exceeds cb_t^o and cb_t^y exceeds μ_t that is, revenue is raised inefficiently.

This argument applies independently of whether the *external* debt service is constrained or not. Constraints on the external debt service do not undermine the government's ability to efficiently raise revenue while constraints on the domestic debt service do. We summarize the discussion in the following proposition:

Proposition 3. An interior choice of r_t^d lets the government equalize the cost-benefit ratio of taxation across cohorts. If r_t^d is in a corner then $\mathrm{cb}_t^o < \mu_t < \mathrm{cb}_t^y$. A domestic return floor undermines the government's ability to efficiently raise revenue, an external return floor does not.

Equilibrium condition (16) conveys two additional important lessons. The first concerns $\mathcal{B}_t^\tau$ as well as $\mathcal{Q}_t^\tau$ and their effect on the cost of taxation and debt returns. Ceteris paribus, positive general equilibrium net benefits of taxation, $\mathcal{B}_t^\tau$, reduce the shadow cost of public funds and thus, work in the direction of higher returns on domestic or external debt, see conditions (14) and (15). At its heart, the increased political support for debt service due to a positive $\mathcal{B}_t^\tau$ derives from a transfer of resources from future, yet unborn generations to current taxpayers that lowers the cost of taxation. If $\mathcal{B}_t^\tau > 0$ then workers are partly "compensated" for their tax payments by the prospect of general equilibrium or policy related net benefits. The mirror image costs of these benefits are born by the future young who are not represented in the political process. Positive general equilibrium effects on the revenue from debt issuance, $\mathcal{Q}_t^\tau$, also work in the direction of lowering the shadow cost of public funds, negative effects in the opposite direction.

The second, related lesson conveyed by condition (16) concerns the equalization of social marginal utilities across cohorts. Although taxes and domestic debt repayment

constitute non-distorting instruments (at the time they are imposed), social marginal utilities are *not* equalized across cohorts, even if the inequality constraint $r_t^d \geq \bar{r}_t^d$ is slack. As conditions (14) and (16) make clear, this is a consequence of the fact that the presence of $\mathcal{B}_t^\tau$ and $\mathcal{Q}_t^\tau$ drives a wedge between the marginal utility of young households and the shadow cost of public funds.[25] As a consequence of this wedge, domestic burden sharing, which is guaranteed as long as the constraint $r_t^d \geq \bar{r}_t^d$ is slack, does not go hand in hand with domestic consumption smoothing.

We summarize the preceding discussion in the following proposition:

Proposition 4. Ceteris paribus, general equilibrium net benefits of taxation, $\mathcal{B}_t^\tau$, and positive general equilibrium effects of taxation on debt sales, $\mathcal{Q}_t^\tau$, reduce the shadow cost of public funds and increase the returns on domestic or external debt. Generically, the presence of $\mathcal{B}_t^\tau$ and $\mathcal{Q}_t^\tau$ drives a wedge between the social marginal utility of young households and the shadow cost of public funds and thus, between the social marginal utilities of young and old households.

4.5 Partial Commitment to Debt Service

When a return floor $\bar{r}_{t+1}^d$ or $\bar{r}_{t+1}^e$ will bind in period $t+1$ then raising it further changes two state variables in the subsequent period: On the one hand, the return floor itself. On the other hand, the capital stock because the binding return floor constitutes a parameter in the household's portfolio choice problem in period t; physical investment is crowded out when a binding, higher return floor $\bar{r}_{t+1}^d$ increases the price at which debt is sold in period t. By affecting two state variables in the subsequent period, the change of a binding return floor triggers general equilibrium effects and may induce policy responses; both have first-order welfare consequences.[26] For example, as stated in proposition 3, a binding $\bar{r}_{t+1}^d$ prevents the government in period $t+1$ from raising revenue in the most cost efficient way.

If the domestic return floor $\bar{r}_{t+1}^d$ can be chosen state by state then the equilibrium conditions are given by

$$\mu_t \nu_t \mathcal{Q}_t^d + \nu_t \mathcal{B}_t^d \leq 0, \quad \bar{r}_{t+1}^d \geq 0 \tag{18}$$

and the complementary slackness condition. Here, $\mathcal{Q}_t^d \equiv \mathrm{d}(q_t^d + q_t^e)/\mathrm{d}\bar{r}_{t+1}^d$ and

$$
\begin{aligned}
\mathcal{B}_t^d \equiv \; & \delta \mathbb{E}_t \left[u'(c_{t+1}^o) \left(k_{t+1} \frac{\mathrm{d}R_{t+1}(k_{t+1})}{\mathrm{d}\bar{r}_{t+1}^d} + q_t^d \frac{\mathrm{d}(r^d(z_{t+1})/q_t^d)}{\mathrm{d}\bar{r}_{t+1}^d} \right) (1 - \tau(z_{t+1})) \right. \\
& + u'(c_{t+1}^o) \left(k_{t+1} R_{t+1}(k_{t+1}) + r^d(z_{t+1}) \right) \frac{\mathrm{d}(1 - \tau(z_{t+1}))}{\mathrm{d}\bar{r}_{t+1}^d} \\
& \left. + v_{t+1}'(g(z_{t+1})) \frac{\mathrm{d}g(z_{t+1})}{\mathrm{d}\bar{r}_{t+1}^d} \right].
\end{aligned}
$$

Note that the first-order condition features an expectation operator although it reflects the consequences of a change of $\bar{r}_{t+1}^d$ in just one state of nature. The reason is that the

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[25]If $\mathcal{B}_t^\tau + \mathcal{Q}_t^\tau u'(c_t^y) = 0$ then the social marginal utilities of old and young households are equalized unless the inequality constraint $r_t^d \geq \bar{r}_t^d$ binds.

[26]In contrast, the welfare consequences of the induced portfolio change of savers are of second order.

change may affect capital accumulation which in turn has consequences across all states of nature in the subsequent period.

Similarly, equilibrium $\bar{r}_{t+1}^e$ in a specific state of nature is characterized by

$$\mu_t \nu_t \mathcal{Q}_t^e + \nu_t \mathcal{B}_t^e \leq 0, \quad \bar{r}_{t+1}^e \geq 0 \tag{19}$$

and the complementary slackness condition, where $\mathcal{Q}_t^e \equiv \mathrm{d}(q_t^d + q_t^e)/\mathrm{d}\bar{r}_{t+1}^e$ and

$$
\begin{aligned}
\mathcal{B}_t^e \quad \equiv \quad & \delta \mathbb{E}_t \left[u'(c_{t+1}^o) \left(k_{t+1} \frac{\mathrm{d}R_{t+1}(k_{t+1})}{\mathrm{d}\bar{r}_{t+1}^e} + q_t^d \frac{\mathrm{d}(r^d(z_{t+1})/q_t^d)}{\mathrm{d}\bar{r}_{t+1}^e} \right) (1 - \tau(z_{t+1})) \right. \\
& \left. + u'(c_{t+1}^o) \left(k_{t+1} R_{t+1}(k_{t+1}) + r^d(z_{t+1}) \right) \frac{\mathrm{d}(1 - \tau(z_{t+1}))}{\mathrm{d}\bar{r}_{t+1}^e} \right. \\
& \left. + v'_{t+1}(g(z_{t+1})) \frac{\mathrm{d}g(z_{t+1})}{\mathrm{d}\bar{r}_{t+1}^e} \right].
\end{aligned}
$$

Note that compared with equilibrium taxes (characterized in condition (16)), equilibrium return floors only have indirect, general equilibrium consequences.

5   Functional Form Assumptions

To characterize politico-economic equilibrium as transparently and compactly as possible we adopt functional form assumptions and derive closed form solutions.

First, we assume that the utility function of households is logarithmic, $u(c_t) = \ln(c_t)$, and the production function Cobb-Douglas, $y_t = a_t k_t^{\alpha_t} \nu_t^{1-\alpha_t}$ with a_t and α_t representing productivity and the capital share in income respectively.[27] Under these assumptions, we can derive closed form solutions for the private sector equilibrium.

Second, we assume that the valuation function of public goods spending is logarithmic as well, $v_t(g_t) \equiv \gamma_t \ln(g_t)$ with $\gamma_t > 0$; this lets us derive closed form solutions for $\mathcal{B}_t^i$ and $\mathcal{Q}_t^i, i = \tau, d, e$.

Finally, we assume that $\varphi_t(r_t^e) = \xi_t \ln(r_t^e)$ with $\xi_t > 0$; that external lenders have logarithmic utility and a discount factor β_t; and that their consumption is given by some power function of after tax income in the domestic economy such that $m_t = \beta_t y_t (1 - \tau_t)/(y_{t+1}(1-\tau_{t+1}))$. The latter assumption, made for tractability, could reflect the fact that external lenders are significantly exposed to income streams from the domestic economy or other, similar economies. The last set of assumptions guarantees closed form solutions for the equilibrium policy choices.

We will first solve for politico-economic equilibrium conditional on a given sequence of return floors, $\{\bar{r}_t^d, \bar{r}_t^e\}$ which we specify as $\bar{r}_t^d = \bar{\rho}(\hat{z}_t)y_t$ and $\bar{r}_t^e = \bar{\sigma}(\hat{z}_t)y_t(1 - \tau_t)$.

5.1   Politico-Economic Equilibrium Conditional on $\bar{\rho}(\cdot)$ and $\bar{\sigma}(\cdot)$

We conjecture that the equilibrium return on domestic and external debt, respectively, is given by $r_t^d = \rho(\hat{z}_t)y_t$ and $r_t^e = \sigma(\hat{z}_t)y_t(1 - \tau_t)$ where $\rho(\cdot)$ and $\sigma(\cdot)$ are arbitrary functions

---

[27]This implies $R_t = \alpha_t a_t k_t^{\alpha_t - 1} \nu_t^{1-\alpha_t}$ and $w_t = (1 - \alpha_t)a_t k_t^{\alpha_t} \nu_t^{-\alpha_t}$.

of the exogenous state. In appendix A, we derive the private sector optimality conditions and the political equilibrium conditions and we verify the conjecture.

The political equilibrium conditions (see conditions (A.3)–(A.6) in appendix A) simplify to

$$g_t = (\omega_t + \nu_t)\gamma_t x(\hat{z}_t)y_t(1 - \tau_t), \tag{20}$$

$$x(\hat{z}_t) \equiv \frac{1 + \rho(\hat{z}_t) + \nu_t \tilde{\mathcal{Q}}_t^\tau}{\omega_t + \nu_t \left(1 + \delta - \tilde{\mathcal{B}}_t^\tau\right)}, \tag{21}$$

$$\rho(\hat{z}_t) = \max\left[\bar{\rho}(\hat{z}_t), -\alpha_t + \frac{\omega_t\left(1 - \alpha_t + \nu_t\tilde{\mathcal{Q}}_t^\tau\right)}{\nu_t\left(1 + \delta - \tilde{\mathcal{B}}_t^\tau\right)}\right], \tag{22}$$

$$\sigma(\hat{z}_t) = \max[\bar{\sigma}(\hat{z}_t), \xi_t x(\hat{z}_t)]. \tag{23}$$

The terms $\tilde{\mathcal{B}}_t^\tau$ and $\tilde{\mathcal{Q}}_t^\tau$ are transformations of the general-equilibrium net benefits of taxation and of the effect of taxation on debt sales, respectively. Both terms are functions of $\hat{z}_t$ only (see appendix A).

The tax rate is pinned down by the budget constraint. Using (20)–(23) as well as the function $\kappa^2(\hat{z}_t)$ defined in appendix A, the budget constraint reduces to

$$(\omega_t + \nu_t)\gamma_t x(\hat{z}_t)y_t(1 - \tau_t) + \rho(\hat{z}_t)y_t + \sigma(\hat{z}_t)y_t(1 - \tau_t) = \tau_t y_t(1 + \rho(\hat{z}_t)) +$$
$$(1 - \alpha_t)y_t(1 - \tau_t)\kappa^2(\hat{z}_t) + \nu_t\beta_t y_t(1 - \tau_t)\mathbb{E}_t[\sigma(\hat{z}_{t+1})]$$

or

$$\frac{\tau_t}{1 - \tau_t} = (\omega_t + \nu_t)\gamma_t x(\hat{z}_t) + \rho(\hat{z}_t) + \sigma(\hat{z}_t) - (1 - \alpha_t)\kappa^2(\hat{z}_t) - \nu_t\beta_t\mathbb{E}_t[\sigma(\hat{z}_{t+1})]. \tag{24}$$

The last two terms on the right-hand side of (24) reflect domestic and external debt sales, respectively. Note that the equilibrium tax rate is a function of $\hat{z}_t$ only; the elasticity of the equilibrium tax function with respect to the capital stock thus equals zero. From (20), the elasticity of the government spending function with respect to the capital stock therefore equals α_t. Note also that for $\omega_t \to \infty$ the tax rate approaches unity.

We have completely characterized politico-economic equilibrium conditional on the functional forms of the return floors, $\bar{\rho}(\cdot)$ and $\bar{\sigma}(\cdot)$:

i. The values $x(\hat{z}_t)$, $\rho(\hat{z}_t)$ and $\sigma(\hat{z}_t)$ for all $\hat{z}_t \in \hat{Z}$ are pinned down by conditions (21)–(23), see appendix A.

ii. The tax rates $\tau(\hat{z}_t)$ then solve condition (24) for all $\hat{z}_t \in \hat{Z}$.

iii. The equilibrium allocation conditional on z_0 is pinned down by condition (20) as well as

$$c_t^o = (\alpha_t + \rho(\hat{z}_t))y_t(1 - \tau(\hat{z}_t)), \tag{25}$$
$$c_t^y = w_t(1 - \tau(\hat{z}_t))/(1 + \delta), \tag{26}$$
$$k_{t+1} = w_t(1 - \tau(\hat{z}_t))\kappa^1(\hat{z}_t), \tag{27}$$
$$q_t^d = w_t(1 - \tau(\hat{z}_t))\kappa^2(\hat{z}_t), \tag{28}$$
$$q_t^e = \beta_t y_t(1 - \tau(\hat{z}_t))\mathbb{E}_t[\sigma_{t+1}(\hat{z}_{t+1})] \tag{29}$$

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for all t and all $\hat{z}_t \in \hat{Z}$ where $\kappa^1(\hat{z}_t)$ and $\kappa^2(\hat{z}_t)$ are defined in appendix A.

5.2 Discussion

To understand the expression for the interior value of $\rho(\hat{z}_t)$ (the second argument of the max operator in (22)) suppose that both the general equilibrium net benefits of higher taxes and the effect of higher taxes on debt purchases equalled zero ($\tilde{\mathcal{B}}_t^\tau = \tilde{\mathcal{Q}}_t^\tau = 0$). Equilibrium condition (22) then would reduce to $\alpha_t + \rho(\hat{z}_t) = \omega_t(1-\alpha_t)/(\nu_t(1+\delta))$, corresponding to the equalization of social marginal utilities across cohorts.[28] An interior equilibrium value for $\rho(\hat{z}_t)$ exceeds the benchmark value supporting inter-generational consumption smoothing when the general equilibrium net benefits of taxation, $\tilde{\mathcal{B}}_t^\tau$, are "sufficiently positive" and the revenue losses on domestic and external debt issuance, $\tilde{\mathcal{Q}}_t^\tau$, "not too negative."

The variable $x(\hat{z}_t)$ in condition (21) represents a benefit-cost ratio of taxes (see (A.6) and (A.7) in appendix A). Its normalized inverse equals the shadow cost of public funds,

$$x(\hat{z}_t)^{-1} = \mu_t y_t(1-\tau_t). \tag{30}$$

The numerator of $x(\hat{z}_t)$ represents the marginal benefit of higher taxation, the marginal gain in government revenue. Normalized by output, this gain equals one (for taxes on labor and capital income) plus $\rho(\hat{z}_t)$ (for taxes on domestic debt service) plus $\nu_t\tilde{\mathcal{Q}}_t^\tau$ (for revenue losses on new debt issuance). The costs, reflected in the denominator of $x(\hat{z}_t)$, derive from the income losses inflicted on the old and the young, including the general equilibrium net benefits. If ρ_t is interior then

$$\mu_t = \frac{\omega_t/(1-\tau_t)}{(\alpha_t+\rho(\hat{z}_t))y_t} = \frac{\nu_t\left(1+\delta-\tilde{\mathcal{B}}_t^\tau\right)/(1-\tau_t)}{\left(1-\alpha_t+\nu_t\tilde{\mathcal{Q}}_t^\tau\right)y_t},$$

corresponding to our earlier finding that $\mu_t = \mathrm{cb}_t^o = \mathrm{cb}_t^y$.

Models in the sovereign debt literature often *assume* that the cost of default is income dependent. Broadly consistent with empirical evidence, these models tend to predict higher average repayment rates on external debt when the domestic economy "booms," and lower repayment rates during domestic "recessions."[29] The model considered here also generates this prediction, $r_t^e \propto y_t$, although it does *not* assume income dependent default costs. Instead, pro-cyclicality of the external debt return derives from the counter-cyclicality of the shadow cost of public funds: when income is low, the cost-benefit ratio of taxation is high and the politically preferred return on external debt low. In addition, the model also predicts pro-cyclical *domestic* debt returns, $r_t^d \propto y_t$, due to the fact that the domestic debt service moves in tandem with the factor incomes of young and old households.

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[28] Equilibrium consumption of old households equals $(\alpha_t+\rho(\hat{z}_t))y_t(1-\tau_t)$ and that of young households $w_t(1-\tau_t)/(1+\delta) = (1-\alpha_t)y_t/\nu_t(1-\tau_t)/(1+\delta)$.

[29] However, see Tomz and Wright (2007) for a critical discussion.

Proposition 5. Under the specified functional form assumptions, the return on external and domestic debt is proportional to domestic income (although the default cost function $\varphi_t(\cdot)$ is independent of income). Income dependence of debt returns reflects the counter-cyclicality of the shadow cost of public funds.

The effect of a marginal tax hike on debt sales consists of two negative components (see appendix A),

$$\nu_t \tilde{\mathcal{Q}}_t^\tau = -(1-\alpha_t)\kappa^2(\hat{z}_t) - \nu_t \beta_t \mathbb{E}_t \left[\sigma(\hat{z}_{t+1})\alpha_{t+1} \right],$$

reflecting the consequences for the prices of domestic and external debt, respectively. Substituting this relationship into condition (21) yields a linear relationship between $x(\hat{z}_t)$, the benefit-cost ratio of taxes, and the terms $\rho(\hat{z}_t) - (1-\alpha_t)\kappa^2(\hat{z}_t)$ which comprise part of the normalized benefit from taxation. Substituting for the latter terms in the equilibrium expression for taxes, (24), implies that the tax wedge can be expressed as a function of $x(\hat{z}_t)$, $\sigma(\hat{z}_t)$, $\sigma(\hat{z}_{t+1})$ and parameters:

$$1 - \tau(\hat{z}_t) =$$
$$\frac{1}{\sigma(\hat{z}_t) - \nu_t \beta_t \mathbb{E}_t[\sigma(\hat{z}_{t+1})(1-\alpha_{t+1})] + x(\hat{z}_t)((\omega_t + \nu_t)(1+\gamma_t) + \nu_t(\delta - \tilde{\mathcal{B}}_t^\tau))}. \tag{31}$$

When the ratio $\sigma(\hat{z}_{t+1})/\sigma(\hat{z}_t)$ is exogenous and $\sigma(\hat{z}_t)$ is interior such that $\sigma(\hat{z}_t) = \xi_t x(\hat{z}_t)$ then the tax wedge is inversely proportional to $x(\hat{z}_t)$; it does not directly depend on a domestic return floor (see also the discussion below).

Finally, an interior domestic debt service implies $g_t/c_t^o = (\omega_t + \nu_t)\gamma_t/\omega_t$; an interior external debt service implies $g_t/r_t^e = (\omega_t + \nu_t)\gamma_t/\xi_t$; and interior domestic and external debt service implies $r_t^e/c_t^o = \xi_t/\omega_t$.

5.3 Steady-State Analysis

When the cardinality of the exogenous state space equals unity, $\#\hat{Z} = 1$, such that the policy instruments are time invariant then the economy converges to a steady state.

In this subsection, we assume that the steady-state external debt service is interior. From (31) and using the definition of $\tilde{\mathcal{B}}_t^\tau$, $\tilde{\mathcal{B}}^\tau = \delta(1 - \alpha(1+\gamma))$, the steady-state tax wedge then equals

$$1 - \tau(\hat{z}) = \frac{1}{\xi(1 - \nu\beta(1-\alpha)) + (1+\gamma)(\omega + \nu(1+\alpha\delta))} \frac{1}{x(\hat{z})} \tag{32}$$

and the product $x(\hat{z})(1 - \tau(\hat{z}))$ only depends on $\alpha, \beta, \gamma, \delta, \nu, \xi$ and ω; it is *independent* of a domestic return floor $\bar{\rho}(\hat{z})$. Using (20), (23), (A.2) and the definition of q^e, the budget constraint (6) then reduces to

$$(\omega+\nu)\gamma X(\hat{z}) + \rho(\hat{z}) + \xi X(\hat{z}) = \tau(\hat{z})(1+\rho(\hat{z})) + (1-\alpha)\frac{\delta}{1+\delta}\frac{\rho(\hat{z})}{\alpha + \rho(\hat{z})}(1-\tau(\hat{z})) + \nu\beta\xi X(\hat{z}),$$

where $X(\hat{z}) \equiv x(\hat{z})(1 - \tau(\hat{z}))$. This implies that in equilibrium,

$$\rho(\hat{z})\left(1 - \frac{\delta}{1+\delta}\frac{1-\alpha}{\alpha + \rho(\hat{z})}\right)(1 - \tau(\hat{z})) - \tau(\hat{z})$$

is a function of parameters only. The term in parentheses multiplying the tax wedge is positive unless the government runs a Ponzi scheme on domestic bond holders. Absent such a Ponzi scheme, the derivative of the same term with respect to ρ is positive as well. A higher domestic return floor, $\bar{\rho}(\hat{z})$, thus increases both the tax rate $\tau(\hat{z})$ and (since $x(\hat{z})(1 - \tau(\hat{z}))$ is constant) the benefit-cost ratio $x(\hat{z})$.

The first denominator on the right-hand side of condition (32) represents normalized (by $yx(1 - \tau)$) net expenditure components in the government's budget constraint. The term ξ represents external debt service; the term $-\xi\nu\beta$ external debt issuance; and the term $\xi\nu\beta\alpha$ the reduction of the debt price due to a tax rise. (The latter term is present because of the substitution for $\rho(\hat{z}_t)$ that led to equation (31).) The term $(\omega + \nu)\gamma$ represents public goods spending; and the term $\omega + \nu + \nu\delta\alpha(1+\gamma)$ reflects the direct and indirect cost of taxation. (Again, this term is present because of the substitution for $\rho(\hat{z}_t)$ that led to equation (31).)

A parameter change that increases (decreases) any of these expenditure components implies that in equilibrium, the factor of proportionality for all net expenditure components, $x(\hat{z})(1 - \tau(\hat{z}))$, must fall (rise). Specifically, a higher capital share, α; stronger preference for public goods, γ; more patience, δ; a higher weight attached to the old, ω; and higher world interest rates, a lower β, all reduce $x(\hat{z})(1 - \tau(\hat{z}))$. The effects of ξ and ν are ambiguous. The sources of these comparative statics results are as follows:

- A higher capital share, α, renders taxation more costly: On the one hand it makes $dq^e/d\tau$ more negative, by changing lenders' asset pricing kernel; and on the other it lowers the general equilibrium net benefit of taxation which depends positively on the elasticity with which a lower capital stock translates into higher interest rates, $1 - \alpha$, and negatively on the elasticity with which a lower capital stock translates into lower public goods spending, α. Both these effects reflect *higher domestic debt service*, because of the substitution for $\rho(\hat{z}_t)$ that led to equation (31).

- A stronger preference for public goods spending, γ, raises the *political demand for spending*. It also lowers the general equilibrium net benefit of taxation by increasing the negative impact on future public goods spending. This goes hand in hand with *higher domestic debt service*, for the reason mentioned above.

- More patience, δ, implies that the political process emphasizes the future net cost of taxation more strongly. Again, this goes hand in hand with *higher domestic debt service*, for the reason mentioned above.

- A stronger weight attached to the old, ω, increases the political *demand for public goods spending* and the cost of reducing the disposable income of the old. Higher costs of taxation go hand in hand with *higher domestic debt service*, for the reason mentioned above.

- A higher world interest rate, lower β, reduces the government revenue raised from *debt issuance*. At the same time it reduces the impact of the negative price effect from higher taxes, reflecting *higher domestic debt service*, for the reason mentioned above. The former effect dominates.

- External debt capacity, ξ, increases the *external debt service* net of new *debt sales* (direct effect and price effect, the latter again reflecting *domestic debt service*, see the discussion of the role of β). As long as ν is not too large (giving rise to a Ponzi game vis-a-vis external lenders) the former effect dominates.

- Finally, population growth, ν, works on the one hand in the same direction as the political weight of the old because the young also care about *public goods provision* and reduced disposable income which, for the reason mentioned earlier, is linked to *domestic debt service*. On the other hand, population growth increases the weight on the general equilibrium net benefit of taxation (which is again connected to *domestic debt service*) and it increases the resources raised by *new debt issuance* (direct effect and price effect, the latter again reflecting *domestic debt service*, in parallel to the effect of β).

  The net effect of ν on $x(\hat{z})(1 - \tau(\hat{z}))$ is negative if $(1 + \alpha\delta)(1 + \gamma) > (1 - \alpha)\beta\xi$ that is, if the external debt capacity (determined by ξ) is relatively small. Intuitively, with little external debt, higher ν mainly affects the government's net expenditures by increasing demand for public goods spending as well as domestic debt service. The factor of proportionality for all net expenditure components, $x(\hat{z})(1 - \tau(\hat{z}))$, therefore falls. If the reverse inequality holds then the revenue effects from external debt issuance dominate. Higher population growth then leads to higher external debt sales and as a consequence, the factor of proportionality for all net expenditure components, $x(\hat{z})(1 - \tau(\hat{z}))$, rises.

From (20) and (23), both g/y and r^e/y are proportional to $x(\hat{z})(1 - \tau(\hat{z}))$. The preceding findings (listed in the first column of table 1) and straightforward calculations therefore immediately imply a series of comparative statics results for the external debt service and public goods spending which are also presented in table 1. Other simple calculations establish that for $\xi \to \infty$, the output share devoted to external debt service approaches $(1 - \nu\beta(1 - \alpha))^{-1}$. Similarly, for $\gamma \to \infty$, the output share devoted to public goods spending approaches $(\omega + \nu)/(\omega + \nu + \nu\alpha\delta)$.[30]

In contrast to the external debt service and public goods spending, the steady-state output share of interior domestic debt service, $\rho(\hat{z})$, is not linear in $x(\hat{z})(1 - \tau(\hat{z}))$. Solving the system of equilibrium conditions yields

$$\rho(\hat{z}) = -\alpha + \frac{(1 - \alpha)\omega \left(1 + \sqrt{1 + \frac{4\alpha\delta(1+\delta)\nu(1+\alpha(\delta(1+\gamma)+\beta\xi))}{(1-\alpha)\omega}}\right)}{2(1 + \delta)\nu(1 + \alpha(\delta(1 + \gamma) + \beta\xi))}.$$

---

[30]The $\nu\alpha\delta$ term reflects the fact that the taxes required to fund current spending depress investment and thus, the tax base for future public goods spending.

	μ^{-1}/y	g/y	r^e/y	r^d/y
α	$-$	$-$	$-$	
β	$+$	$+$	$+$	$-$
γ	$-$	(c)	$-$	$-$
δ	$-$	$-$	$-$	(f)
$\bar{\rho}$	0	0	0	$+$
ξ	(a)	(a)	$+$	$-$
ω	$-$	(d)	$-$	$+$
ν	(b)	(e)	(b)	$-$

Table 1: Comparative statics in steady state.

Note: Cells in the table indicate the signs of the derivatives of the variables μ/y, g/y, r^e/y and r^d/y with respect to the model parameters. Results hold under the assumption that σ is interior. (a) Derivative is negative if $1 > \nu\beta(1-\alpha)$, and positive otherwise. (b) Derivative is negative if $(1+\alpha\delta)(1+\gamma) > (1-\alpha)\beta\xi$, and positive otherwise. (c) Derivative is positive if $\xi(1 - \nu\beta(1-\alpha)) + \omega + \nu(1+\alpha\delta) > 0$, and negative otherwise. (d) Derivative is positive if $\alpha\delta\nu(1+\gamma) + \xi(1 - \beta\nu(1-\alpha)) > 0$, and negative otherwise. (e) Derivative is positive if $\xi(1 + \omega\beta(1-\alpha)) > \omega\alpha\delta(1+\gamma)$, and negative otherwise. (f) Sign of derivative depends on more complicated parameter condition. A "0" indicates a derivative equal to zero.

For $\omega \to 0$, $\rho(\hat{z})$ approaches $-\alpha$ that is, retirees are fully expropriated: the government confiscates the full return on capital and does not pay any return on domestic bonds. For $\beta = \delta = 0$ (implying $\mathcal{B}^\tau = \mathcal{Q}^\tau = 0$), $\rho(\hat{z})$ reduces to $-\alpha + \omega(1-\alpha)/\nu$: the consumption of old and young households is equalized, controlling for differences in political influence. The comparative statics of $\rho(\hat{z})$ with respect to the model parameters mostly are monotone; the comparative statics results are collected in table 1. Most importantly, $\rho(\hat{z})$ increases in ω and decreases in ν.

We summarize the main results in the following proposition:

Proposition 6. Under the specified functional form assumptions and if $\sigma(\hat{z})$ is interior, the steady-state values of $x(\hat{z})(1 - \tau(\hat{z}))$ (and thus, μy) as well as g/y and r^e/y are invariant to $\bar{\rho}(\hat{z})$. Absent a Ponzi scheme on domestic bond holders, the steady-state tax rate increases in $\bar{\rho}(\hat{z})$. Table 1 collects various comparative statics results.

Proposition 6 implies that *conditional on output*, a binding domestic return floor imposes a burden on taxpayers but not on external lenders or recipients of public goods. However, output does respond to taxes and a binding commitment to domestic debt service affects steady-state public goods spending and external debt service through its impact on capital accumulation and output.

Example: Demographic Ageing According to proposition 6 demographic ageing (a reduction of ν) increases r^d/y but has an ambiguous effect on g/y and r^e/y. Since ageing constitutes one of the most important changes in the economic environment that

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Parameter	Value
a	1.0000
α	0.3000
β	0.7220
γ	0.3056
δ	0.7220
ξ	0.4849
ω	1.1831
ν	1.2000

Table 2: Calibration.

has occurred over the last decades, we investigate further how such a change affects the endogenous model variables. We do so by means of a numerical example, adopting the parameter values reported in table 2 which are motivated as follows: The value for α is standard; the value for β and δ is taken from Gonzalez-Eiras and Niepelt (2012) and corresponds to 0.9892^{30}; and the values for ω, γ and ξ are chosen to match the moments $\rho = \sigma(1-\tau)$, $\tau = 0.25$ and $g/y = 0.2$ when $\nu = 1.2$ (corresponding to the projected population growth rate over thirty years in Europe in the year 2020). With these parameter values, demographic ageing weakly decreases g/y while it increases both r^e/y and r^d/y.

Figure 3 illustrates the effect of ν on the spending components as well as other policy instruments and macroeconomic outcomes. The solid lines indicate outcomes when $\bar{\rho}(\hat{z}) = \bar{\sigma}(\hat{z}) = 0$ (no return floors) while the dashed and dotted lines, respectively, indicate outcomes with a domestic or external return floor ($\bar{\rho}(\hat{z}) = 0.15$ or $\bar{\sigma}(\hat{z}) = 20$). Absent return floors, demographic ageing increases the relative importance of domestic relative to external debt. While domestic debt returns strongly increase and domestic debt issuance rises (although with lower ν, there is less scope to issue debt), external debt returns increase only slowly and external debt issuance actually falls.

While debt returns rise with ageing, public goods spending decreases only by a small amount (see the right-most panel in the bottom row of the figure). As a consequence, demographic ageing is accompanied by rising tax rates and an increase in the shadow value of public funds (see the fourth and second panel in the middle row, respectively). Output per capita of the old falls, for two reasons. First, a smaller workforce per capita of the old reduces output given capital. Second, a higher tax rate reduces savings out of (higher) wages; and the increased return on domestic debt raises the share of savings funding debt purchases, crowding out physical capital accumulation (see the left panels in the middle and bottom row of the figure).

The last row of the figure displays the evolution of investment, consumption of young and old households, and public good purchases. These four components do *not* sum to unity because consumption of the young is represented in terms of per-capita of the young
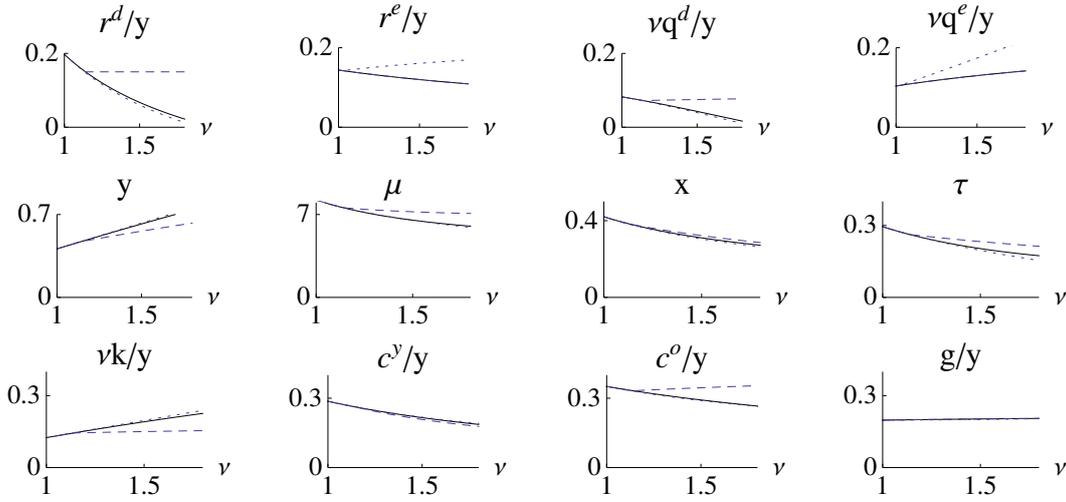
Figure 3: Effects of population growth, ν, in steady state. Solid, dashed and dotted lines, respectively, indicate outcomes with no return floor, a domestic return floor, and an external debt floor.

(rather than the old) and because the economy's capital account is open. At $\nu = 1.2$, the net flows from the economy to foreigners, $r^e - \nu q^e$, amount to 1.8 percent of output. Note that this is the case *although* $\delta = \beta$. Unlike representative agent models of sovereign debt that typically assume large autarky interest rates $(\delta \ll \beta)$ to generate sovereign debt issuance, in the overlapping generations framework considered here sovereign borrowing is sustained even if δ equals β.

The dashed and dotted lines illustrate the effect of return floors for domestic and external debt, respectively. Since debt returns rise with population ageing, return floors only bind when the population growth rate is high. Note that for any value of ν, the product μy (or equivalently, $x(1-\tau)$) is identical in the case without return floors (solid lines) and the case with a domestic return floor (dashed lines). This confirms proposition 6. With an external debt floor (dotted lines), more external debt can be issued, the tax rate is lowered, and output is slightly higher. Note that in line with proposition 3, an external return floor only has a negligible effect on the shadow value of public funds, μ, in contrast to a domestic return floor.

The main driver of the higher domestic debt return in response to ageing is the fall in capital income kR (the wage remains roughly constant). Payments to domestic debt holders dampen the implications of this change of relative factor incomes. But the dampening effect substantially differs from what a "naive" perspective would suggest, see figure 4. The solid lines in the figure indicate, from left to right, the actual values of r^d/y, r^e/y, g/y and τ, respectively. The dashed lines indicate the outcomes when equations (20)–(23) are solved subject to all $\tilde{\mathcal{B}}^\tau$ terms set equal to zero that is, if the general equilibrium net benefits of taxation remain unaccounted for. Consistent with proposition 4 taxes are lower in this case, reflecting the fact that the general equilibrium net benefits of taxation are positive. As a consequence, spending on debt returns and public goods provision is
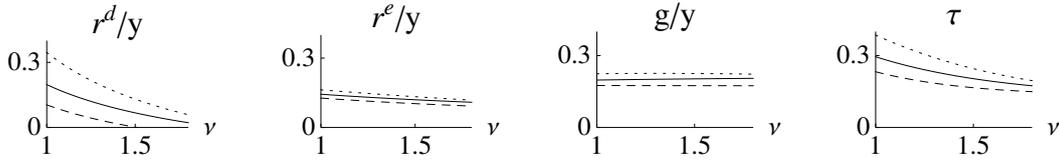
Figure 4: Effects of population growth, ν, in steady state: Actual and "naive" outcomes. Solid, dashed and dotted lines, respectively, indicate actual outcomes, hypothetical outcomes "without $\tilde{\mathcal{B}}^\tau$," and hypothetical outcomes "without $\tilde{\mathcal{B}}^\tau$ and $\tilde{\mathcal{Q}}^\tau$."

lower as well.

The dotted lines in the figure indicate the outcomes when the equilibrium conditions are solved subject to all $\tilde{\mathcal{B}}^\tau$ and $\tilde{\mathcal{Q}}^\tau$ terms set equal to zero that is, if the general equilibrium net benefits of taxation *and* the implications of taxation for debt prices are not accounted for.[31] Since a tax hike reduces the revenue from debt issuance, not internalizing this effect leads the political process to levy higher taxes, and to spend more. As the figure shows, $\tilde{\mathcal{B}}^\tau$ and $\tilde{\mathcal{Q}}^\tau$ have stronger effects on taxes and domestic debt returns than on external debt returns and public goods purchases in equilibrium.

5.4 Dynamic Analysis

Next, we introduce dynamic and stochastic elements in the analysis. We consider a series of numerical examples and adopt the baseline parameterization reported in table 2. For simplicity, we assume that there are just two states of nature, $\#\hat{Z} = 2$; enlarging the state space is computationally essentially costless since the equilibrium outcomes have been solved for in closed form. Unless otherwise noted, we assume that all elements of the Markov transition matrix are 0.5.

Example: Exogenous Sovereign Risk We first consider the effects of exogenous variation in external debt capacity that is, variation of ξ_t. We assume that ξ_t randomly fluctuates around the steady-state value reported in table 2, taking the values $0.4849 - 0.1$ or $0.4849 + 0.1$. Figure 5 illustrates the effects. On the left-hand side, the figure displays the equilibrium values when the low ξ_t-value is realized, and on the right-hand side it illustrates outcomes when ξ_t is high. The figure displays both the actual equilibrium realizations in the stochastic example (connected by solid lines) and the hypothetical steady-state outcomes under the assumption that ξ_t is either always low or always high (points are connected by dotted lines). The shaded area between pairs of solid and dotted lines therefore indicates how strongly the stochastic and the steady-state outcomes differ.

Public goods spending, domestic debt returns and domestic debt prices barely differ across states and are very similar to the corresponding hypothetical steady-state outcomes. External debt returns across the two states also parallel the hypothetical steady-state

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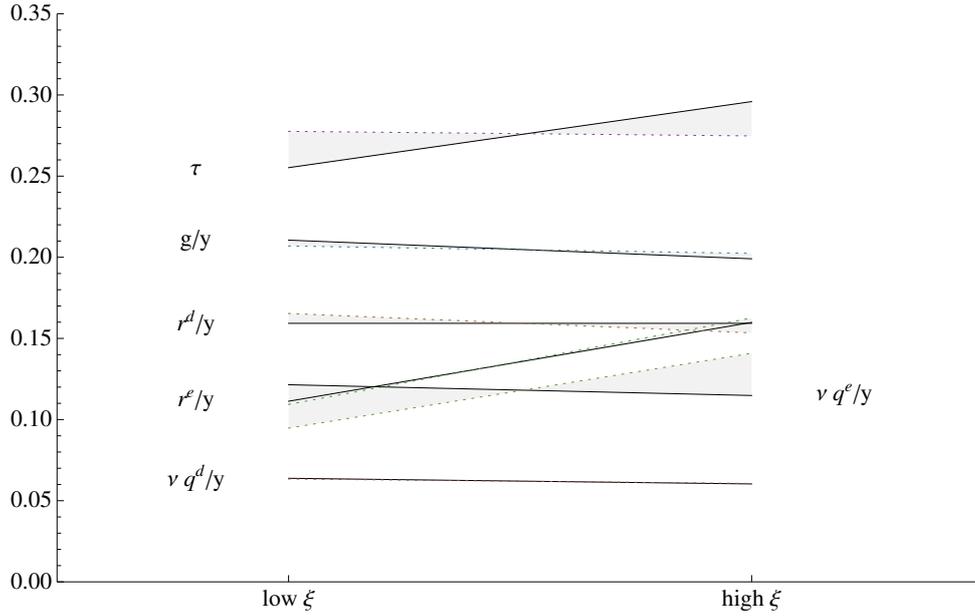[31]In this case, we have $r^d/y = -\alpha + (1-\alpha)/\nu \cdot \omega/(1 + \alpha\delta(1+\gamma))$.

Figure 5: Effects of risky external debt capacity, ξ_t. In addition to the equilibrium realizations of the variables in the two states (connected by solid lines) the figure displays the corresponding steady-state outcomes under the assumption that either state is permanent (connected by dotted lines; the area between pairs of solid and dotted lines is shaded).

outcomes but these returns strongly vary across states, as to be expected: the higher is ξ_t, the higher the return. In steady state, these differences would be reflected in different prices (see the dotted line for $\nu q^e/y$). In the stochastic environment, in contrast, the external debt price is roughly constant across states, reflecting the average return. Since external debt returns thus are state contingent but debt sales and public goods purchases barely are, taxes are state contingent as well. In effect, the political process engages in the opposite of tax smoothing: it adjusts tax rates to accommodate debt returns.

Example: Inequality Shocks Next, we consider the effects of inequality shocks due to variation of the capital income share, α_t. We assume that α_t randomly fluctuates between $0.3 - 0.05$ and $0.3 + 0.05$. Figure 6 illustrates the effects. Similar to what would happen in steady state, higher α_t gives rise to lower domestic debt returns because the political process aims at partly compensating factor income differentials. Unlike in the corresponding hypothetical steady states, the variation in debt returns is not reflected in debt prices at issuance, domestic debt prices rather reflect average returns. Taxes "absorb" the fluctuation in r^d/y given that the other variables barely vary across states.

Example: Random Wars The previous two examples paint a picture of debt returns that strongly vary with the state of nature. It is natural to ask, then, whether the government can insure against government spending shocks in politico-economic equilibrium that is, whether debt returns can be contingent on the realization of γ_t in equilibrium.
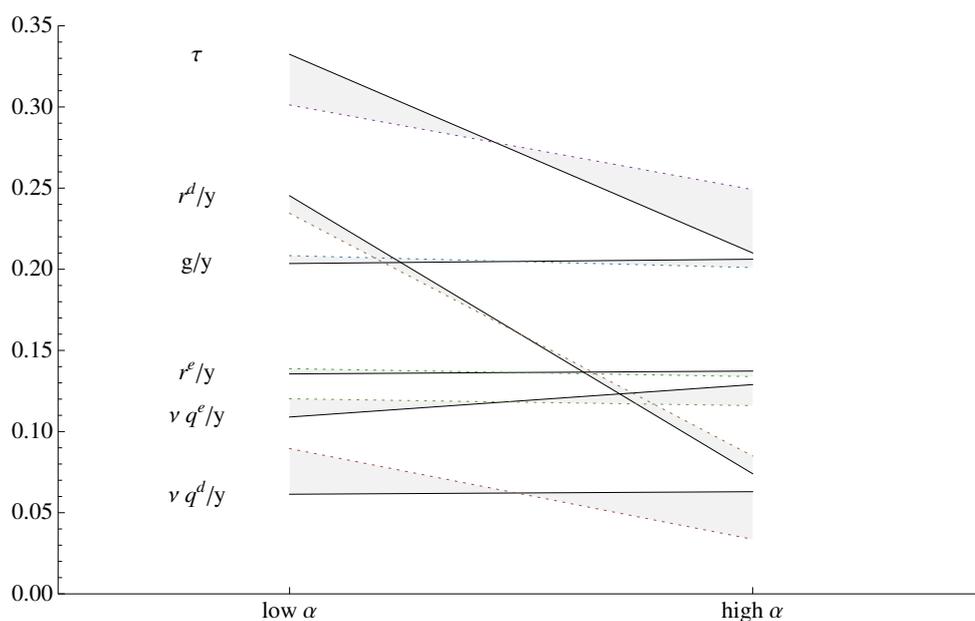
Figure 6: Effects of risky capital income share, α_t. See explanations to previous figure.

In the spirit of one of the scenarios considered by Lucas and Stokey (1983) we analyze the consequences of a small risk of a large "war." Specifically, we assume that with a probability of 5 percent, γ_t doubles in size relative to the baseline value of 0.3056; once high, γ_t reverts to its baseline value with a 95 percent probability.

Figure 7 illustrates the effects: Risk sharing between the government and the private sector is very limited. Variation in γ_t is reflected in time varying public goods spending ("defense outlays") and taxes. But both the return and the sale of (domestic and external) debt barely vary across states. They are also very similar to what they would be in steady state. Intuitively, rendering the return to debt more strongly state contingent would imply that the wealth distribution across generations (in the case of domestic debt) or across creditor groups (in the case of external debt) would become more strongly state contingent as well, running counter to the objective pursued by the political process. Subject to the requirement of a balanced wealth distribution, taxes rather than state-contingent debt returns offer the preferred means to transfer resources from the private to the public sector. As a consequence, the normative tax smoothing prescription in the context of distorting taxes is again put on its head since distributive motivations rule.

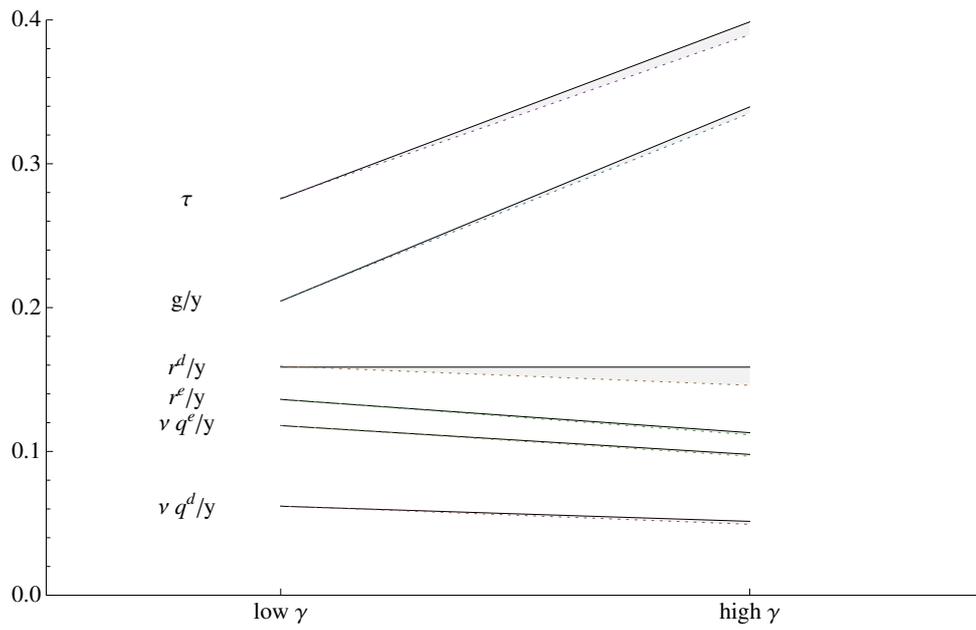## 5.5 Equilibrium Partial Commitment

TBC

Figure 7: Effects of risky public goods needs, γ_t. See explanations to previous figure.

6   Conclusions

TBC

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A   Derivation of Equilibrium for Given $\bar{\rho}(\cdot)$ and $\bar{\sigma}(\cdot)$

A.1   Private Sector Equilibrium

With logarithmic utility the condition for optimal savings, (12), reduces to

$$\frac{k_{t+1} + q_t^d}{w_t(1 - \tau_t) - (k_{t+1} + q_t^d)} = \delta.$$

From the budget constraint, (2), it follows that

$$c_t^y = \frac{1}{1+\delta} w_t(1 - \tau_t) \quad \text{and} \quad k_{t+1} + q_t^d = \frac{\delta}{1+\delta} w_t(1 - \tau_t).$$

The portfolio composition is pinned down by the Euler equations. We conjecture, and later verify, that equilibrium domestic debt service satisfies $r_t^d = \rho_t y_t$ where ρ_t is a function of $\hat{z}_t$ only, $\rho_t \equiv \rho(\hat{z}_t)$. (If the return floor is binding, then $\rho(\hat{z}_t) = \bar{\rho}(\hat{z}_t)$.) Under this conjecture consumption of an old household satisfies, from (3),

$$c_{t+1}^o = (k_{t+1} R_{t+1} + r_{t+1}^d)(1 - \tau_{t+1}) = (\alpha_{t+1} + \rho_{t+1}) y_{t+1}(1 - \tau_{t+1}).$$

The first-order condition with respect to capital, (9), then reduces to

$$\frac{1+\delta}{w_t(1 - \tau_t)} = \delta \mathbb{E}_t\left[\frac{R_{t+1}}{(\alpha_{t+1} + \rho_{t+1}) y_{t+1}}\right] = \delta \mathbb{E}_t\left[\frac{\alpha_{t+1}}{(\alpha_{t+1} + \rho_{t+1}) k_{t+1}}\right],$$

implying

$$k_{t+1} = w_t(1 - \tau_t)\kappa^1(\hat{z}_t), \quad \kappa^1(\hat{z}_t) \equiv \frac{\delta}{1+\delta} \mathbb{E}_t\left[\frac{\alpha_{t+1}}{\alpha_{t+1} + \rho_{t+1}}\right], \tag{A.1}$$

where we use the Markov structure of $\hat{z}_t$. Similarly, the first-order condition for domestic debt, (10), implies

$$q_t^d = w_t(1 - \tau_t)\kappa^2(\hat{z}_t), \quad \kappa^2(\hat{z}_t) \equiv \frac{\delta}{1+\delta} \mathbb{E}_t\left[\frac{\rho_{t+1}}{\alpha_{t+1} + \rho_{t+1}}\right]. \tag{A.2}$$

The share of workers' disposable income invested in capital, $\kappa^1(\hat{z}_t)$, depends positively on the anticipated capital share. The remaining share of savings is invested in debt.

The equilibrium gross rate of return on domestic bonds equals

$$\frac{r_{t+1}^d}{q_t^d} = \frac{\rho_{t+1} y_{t+1}}{w_t(1 - \tau_t)\kappa^2(\hat{z}_t)} = \frac{\rho_{t+1} y_{t+1}\kappa^1(\hat{z}_t)}{k_{t+1}\kappa^2(\hat{z}_t)} = \frac{\rho_{t+1}\kappa^1(\hat{z}_t)}{\alpha_{t+1}\kappa^2(\hat{z}_t)} R_{t+1}.$$

If z_{t+1} can only take one value then this expression reduces to R_{t+1}—bonds and capital are perfect substitutes.

A.2 General-Equilibrium Net Benefits

The general equilibrium net benefit of taxation, $\mathcal{B}_t^\tau$, comprises three effects: By changing capital accumulation, taxes affect the return on physical capital; the return on domestic debt; and future policy. We consider these effects in turn.

The welfare effect of a higher induced return on capital equals

$$
\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\tau_t}\frac{\mathrm{d}R_{t+1}}{\mathrm{d}k_{t+1}}\delta u'(c_{t+1}^o)k_{t+1}(1-\tau_{t+1})
$$

$$
= -w_t\kappa^1(\hat{z}_t)\alpha_{t+1}(\alpha_{t+1}-1)a_{t+1}k_{t+1}^{\alpha_{t+1}-2}\nu_{t+1}^{1-\alpha_{t+1}}\frac{\delta k_{t+1}}{(\alpha_{t+1}+\rho_{t+1})y_{t+1}}
$$

$$
= -\frac{w_t\kappa^1(\hat{z}_t)}{k_{t+1}}(\alpha_{t+1}-1)\frac{\alpha_{t+1}\delta}{\alpha_{t+1}+\rho_{t+1}} = -\frac{1}{1-\tau_t}(\alpha_{t+1}-1)\frac{\alpha_{t+1}\delta}{\alpha_{t+1}+\rho_{t+1}}.
$$

Similarly, using the relation between R_{t+1} and r_{t+1}^d/q_t^d, the welfare effect of a changed return on domestic debt equals

$$
\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\tau_t}\frac{\mathrm{d}(r_{t+1}^d/q_t^d)}{\mathrm{d}k_{t+1}}\delta u'(c_{t+1}^o)q_t^d(1-\tau_{t+1}) = -\frac{1}{1-\tau_t}(\alpha_{t+1}-1)\frac{\rho_{t+1}\delta}{\alpha_{t+1}+\rho_{t+1}}.
$$

The welfare effect due to induced changes in the tax policy is given by

$$
\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\tau_t}\frac{\mathrm{d}(1-\tau_{t+1})}{\mathrm{d}k_{t+1}}\delta u'(c_{t+1}^o)(k_{t+1}R_{t+1}+r_{t+1}^d)
$$

$$
= -w_t\kappa^1(\hat{z}_t)\frac{\mathrm{d}(1-\tau_{t+1})}{\mathrm{d}k_{t+1}}\frac{\delta}{1-\tau_{t+1}} = -\frac{1}{1-\tau_t}\delta\eta_{k(1-\tau),t+1},
$$

where $\eta_{k(1-\tau),t+1}$ denotes the equilibrium elasticity of the tax-wedge with respect to capital. Finally, the welfare effect due to induced changes in government spending equals

$$
\frac{\mathrm{d}k_{t+1}}{\mathrm{d}\tau_t}\frac{\mathrm{d}g_{t+1}}{\mathrm{d}k_{t+1}}\delta v'_{t+1}(g_{t+1}) = -\frac{w_t\kappa^1(\hat{z}_t)}{k_{t+1}}\frac{\mathrm{d}g_{t+1}}{\mathrm{d}k_{t+1}}\frac{\delta\gamma_{t+1}k_{t+1}}{g_{t+1}} = -\frac{1}{1-\tau_t}\delta\gamma_{t+1}\eta_{kg,t+1}
$$

where $\eta_{kg,t+1}$ denotes the equilibrium elasticity of government spending with respect to the capital stock.[32]

In sum, the general equilibrium net benefit of taxation equals

$$
\mathcal{B}_t^\tau = \frac{1}{1-\tau_t}\delta\mathbb{E}_t[1-\alpha_{t+1}-\eta_{k(1-\tau),t+1}-\gamma_{t+1}\eta_{kg,t+1}].
$$

Intuitively, depressing capital accumulation by raising contemporaneous taxes yields a benefit for young voters because it increases the interest rate on capital and debt; this effect equals $\delta\mathbb{E}_t[(1-\alpha_{t+1})/(1-\tau_t)]$. But it also yields a cost if lower capital accumulation leads to lower government spending or higher taxes; this cost equals $\delta(\eta_{k(1-\tau),t+1}+\gamma_{t+1}\eta_{kg,t+1})/(1-\tau_t)$ in size.

---

[32]We conjecture, and later verify, that the elasticities of the policy functions are orthogonal to the capital stock. If this were not the case, one might interpret $\eta_{k(1-\tau),t+1}$ and $\eta_{kg,t+1}$ as first-order approximations.

A.3   Politico-Economic Equilibrium

We conjecture, and later verify, that equilibrium external debt service satisfies $r_t^e = \sigma_t y_t(1 - \tau_t)$ where σ_t is a function of $\hat{z}_t$ only, $\sigma_t \equiv \sigma(\hat{z}_t)$. (If the return floor is binding, then $\sigma(\hat{z}_t) = \bar{\sigma}(\hat{z}_t)$.) Accordingly, we have

$$q_t^e = \mathbb{E}_t[m_t \sigma(\hat{z}_{t+1}) y_{t+1}(1 - \tau_{t+1})] = \beta_t y_t(1 - \tau_t) \mathbb{E}_t[\sigma(\hat{z}_{t+1})]$$

and[33]

$$\frac{dq_t^e}{d\tau_t} = -\beta_t y_t \mathbb{E}_t \left[\sigma(\hat{z}_{t+1})(\alpha_{t+1} + \eta_{k(1-\tau),t+1}) \right].$$

This implies

$$\nu_t \mathcal{Q}_t^\tau = -y_t \left((1 - \alpha_t) \kappa^2(\hat{z}_t) + \nu_t \beta_t \mathbb{E}_t \left[\sigma(\hat{z}_{t+1})(\alpha_{t+1} + \eta_{k(1-\tau),t+1}) \right] \right).$$

The optimality conditions (13)–(16) now reduce to

$$(\omega_t + \nu_t)\frac{\gamma_t}{g_t} = \mu_t, \tag{A.3}$$

$$\omega_t \frac{1}{(\alpha_t + \rho(\hat{z}_t)) y_t (1 - \tau_t)} \leq \mu_t, \quad \rho(\hat{z}_t) \geq \bar{\rho}(\hat{z}_t), \tag{A.4}$$

$$\xi_t \frac{1}{\sigma(\hat{z}_t) y_t (1 - \tau_t)} \leq \mu_t, \quad \sigma(\hat{z}_t) \geq \bar{\sigma}(\hat{z}_t), \tag{A.5}$$

$$\frac{\omega_t + \nu_t(1 + \delta)}{1 - \tau_t} - \nu_t \mathcal{B}_t^\tau = \mu_t \left(y_t (1 + \rho(\hat{z}_t)) + \nu_t \mathcal{Q}_t^\tau \right). \tag{A.6}$$

Recall that $\mathcal{B}_t^\tau$ is proportional to $(1 - \tau_t)^{-1}$ and $\nu_t \mathcal{Q}_t^\tau$ is proportional to y_t.

In representing these first-order conditions we use the earlier conjecture that the return on domestic debt is proportional to output. This conjecture is now verified: If (A.4) holds with equality then (A.4) and (A.6) imply that ρ_t indeed is a function of $\hat{z}_t$ only—as is $\mu_t y_t(1 - \tau_t)$. If (A.4) holds with inequality, in contrast, then ρ_t and $\mu_t y_t(1 - \tau_t)$ still are functions of $\hat{z}_t$ only, due to our assumption that $\bar{r}_t^d \equiv \bar{\rho}(\hat{z}_t) y_t$. Our second conjecture, that the return on external debt is proportional to after-tax output, also is verified, by a parallel argument that uses (A.5) and (A.6). The exact expressions for $\rho(\hat{z}_t)$ and $\sigma(\hat{z}_t)$ are yet to be determined, however.

---

[33]Using

$$
\begin{aligned}
\frac{d(y_{t+1}(1 - \tau_{t+1}))}{d\tau_t} &= \frac{dk_{t+1}}{d\tau_t} \frac{d(y_{t+1}(1 - \tau_{t+1}))}{dk_{t+1}} = -\frac{k_{t+1}}{1 - \tau_t} \left((1 - \tau_{t+1}) \frac{dy_{t+1}}{dk_{t+1}} + y_{t+1} \frac{d(1 - \tau_{t+1})}{dk_{t+1}} \right) \\
&= -\frac{k_{t+1}}{1 - \tau_t} \left((1 - \tau_{t+1}) \alpha_{t+1} \frac{y_{t+1}}{k_{t+1}} + y_{t+1} \frac{d(1 - \tau_{t+1})}{dk_{t+1}} \right),
\end{aligned}
$$

it follows that

$$\frac{dq_t^e}{d\tau_t} = \beta_t \mathbb{E}_t \left[\frac{y_t(1 - \tau_t)\sigma(\hat{z}_{t+1})}{y_{t+1}(1 - \tau_{t+1})} \frac{d(y_{t+1}(1 - \tau_{t+1}))}{d\tau_t} \right] = -\beta_t y_t \mathbb{E}_t \left[\sigma(\hat{z}_{t+1})(\alpha_{t+1} + \eta_{k(1-\tau),t+1}) \right].$$

Equation (A.6) can be expressed as

$$\mu_t y_t(1-\tau_t) = \frac{\omega_t + \nu_t\left(1+\delta - \tilde{\mathcal{B}}_t^\tau\right)}{1 + \rho(\hat{z}_t) + \nu_t\tilde{\mathcal{Q}}_t^\tau}, \tag{A.7}$$

where

$$
\begin{aligned}
\tilde{\mathcal{B}}_t^\tau &\equiv \mathcal{B}_t^\tau(1-\tau_t) = \delta\mathbb{E}_t[1 - \alpha_{t+1} - \eta_{k(1-\tau),t+1} - \gamma_{t+1}\eta_{kg,t+1}], \\
\nu_t\tilde{\mathcal{Q}}_t^\tau &\equiv \frac{\nu_t\mathcal{Q}_t^\tau}{y_t} = -(1-\alpha_t)\kappa^2(\hat{z}_t) - \nu_t\beta_t\mathbb{E}_t\left[\sigma(\hat{z}_{t+1})(\alpha_{t+1} + \eta_{k(1-\tau),t+1})\right].
\end{aligned}
$$

Conditions (A.3)–(A.5) simplify to (20)–(23) in the text.

A.4 Solving the Equilibrium Conditions

With $\eta_{kg,t+1} = \alpha_{t+1}$ and $\eta_{k(1-\tau),t+1} = 0$ (see the discussion in the text), we have

$$\tilde{\mathcal{B}}_t^\tau = \delta\mathbb{E}_t[1 - \alpha_{t+1}(1 + \gamma_{t+1})].$$

The remaining equilibrium conditions are solved in three steps. First, we simultaneously solve (21)–(23) as well as (A.2) and

$$\nu_t\tilde{\mathcal{Q}}_t^\tau = -(1-\alpha_t)\kappa^2(\hat{z}_t) - \nu_t\beta_t\mathbb{E}_t\left[\sigma(\hat{z}_{t+1})\alpha_{t+1}\right].$$

Second, we solve (24) and (A.1). Finally, conditional on the initial state z_0 the equilibrium allocation follows from (20) as well as (25)–(29).

If the cardinality of the exogenous state space $\hat{Z}$ equals unity then the system to be solved in the first step can be solved in closed form. If the cardinality exceeds unity linearizing the right-hand side of equation (A.2) renders the full system to be solved in the first step linear.