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Buffer stocks and precautionary savings with loss aversion

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Abstract

This paper reexamines buffer stocks and precautionary savings in the presence of loss aversion. We assume that agents are disappointment averse, as in Gul [Econometrica, 59 (1991) 667–686]. We show that the concavity of the marginal utility continues to determine precautionary saving, but its effect is of a *second order* magnitude (proportional to the square of the coefficient of variation) compared to the *first order* effect (proportional to the coefficient of variation) induced by loss aversion. We show that a stabilization fund that is rather small when agents are maximizing the conventional expected utility, turns out to be rather large with loss aversion. © 1998 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The purpose of this paper is to show that loss aversion may lead to a much larger optimal size of stabilization funds and precautionary savings than the size predicted if agents are maximizing the conventional expected utility. Loss aversion is the tendency of agents to be more sensitive to reductions in their consumption than to increases relative to a reference point. It characterizes many of the recent generalized expected utility frameworks. Specifically, we investigate the impact of volatility

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on the optimal buffer stock and on precautionary saving in a disappointment aversion utility framework, as postulated by Gul (1991). His approach provides an axiomatic extension of the expected utility approach to account for the Allias paradox. This is done by modeling an agent who maximizes a weighted sum of utility, where the weights deviate from the probabilities so as to reflect disappointment aversion. Gul's specification is a convenient framework as it encompasses the expected utility approach as a special case, allowing one to analyze the impact of disappointment aversion in comparison to the case where agents are maximizing a conventional expected utility.

The interest in this approach stems from several observations. First, there is a significant body of controlled experiments that supports the need to model behavior using the generalized expected utility framework.[1] Second, the results of the generalized-expected utility approach are in contrast with the predictions of the neoclassical approach, where the impact of volatility on precautionary saving is found to be ambiguous, and typically of a second order magnitude (i.e. proportional to the square of the coefficient of variation).[2] Third, developing countries are managing actively various stabilization funds of one sort or another. These funds range from international reserves managed by central banks, to Commodity Stabilization Funds (CSF henceforth) and buffer stocks.[3,4] The impact of volatility on the demand for these funds has been the focus of significant theoretical research. Various studies identified rules for maximizing expected utility agents who manage stabilization funds and buffer stocks.[5] A frequent conclusion is that the gains from the optimal management of stabilization funds and buffer stocks are rather small. For example, Newbery and Stiglitz (1981, p. 420) concluded that 'even for large risk and low storage costs the average buffer stock is small, and that the buffer rule which would be optimal if costs are ignored often looks relatively unattractive compared with alternatives once the costs are included'. Similar results have been obtained by Deaton (1991) who studied precautionary savings in a model where agents are liquidity constrained.[6]

The practice of various developing countries suggests that most manage a sizable stock of international reserves, and some manage large CSFs. An example of a

[1] For example, see the evidence reported by Kahneman and Tversky (1979) and Tversky and Kahneman (1991). Epstein (1992) and Harless and Camerer (1994) provide a useful assessment and further references. In concluding their paper Harless and Camerer (1994) pointed out that 'The pairwise-choice studies suggest that violations of expected utility are robust enough that modeling of aggregate economic behavior based on alternatives to expected utility is well worth exploring' (p. 1286).

[2] Further discussion on volatility in a non-expected utility framework can be found in Segal and Spivak (1990) and Epstein (1992).

[3] Chile, Colombia, Oman and Papua new Guinea have had stabilization funds for some time. Many oil exporting countries established some type of a fund with the additional oil income stemming from the period of the Gulf war (e.g. Ecuador, Mexico, Nigeria and Venezuela). For further details see Arrau and Claessens (1992).

[4] Various studies found a well defined demand for international reserves, a demand whose functional form is similar among countries choosing different exchange rate regimes (for example, see Frenkel, 1974; Edwards, 1983).

[5] See Newbery and Stiglitz, 1981; Deaton, 1991; Basch and Engel, 1993; Hausmann et al., 1993.

stabilization fund that is frequently praised as a success story is the Chilean Copper Stabilization Fund. Applying Deaton's framework, Arrau and Claessens (1992) found that the optimal size of the Chilean fund is very small (about US$ 70 million, roughly 20% of copper exports).[7] In fact, the size of the *actual* fund is much larger (more than US$ 1.8 billion that year).[8] The possible merits of a stabilization fund are of obvious concern to countries which specialize in exporting few primary products. In these circumstances terms of trade volatility translate into volatility of fiscal revenue.[9]

After a brief overview of Gul's framework, we derive the risk premium for a disappointment averse agent. We point out that while volatility induces second order effects on the risk premium if agents are maximizing expected utility, volatility induces first order effects on the risk premium when agents are disappointment averse. This outcome has a direct bearing on all the analytical results characterizing demands for assets as a function of the risk premium. We illustrate this point by extending a model advanced by Newbery and Stiglitz (1981) who characterized the optimal buffer stock for a risk averse agent. Our analysis extends this model for the case where agents are disappointment averse, showing that a stabilization fund that is rather small when agents are maximizing the conventional expected utility, turns out to be rather large when agents are disappointment averse.

The issue of the optimal CSF has many other aspects that are ignored by our paper. For example, in weak political systems a large CSF may be unfeasible due to rent seeking activities — any liquid funds may be expropriated by pressure groups maximizing their narrow agenda.[10] On the other hand, significant adjustment costs to changes in government consumption would increase the optimal size of the CSF.[11] Hence, the issue of the usefulness of CSF hinges upon various considera-

[6] Deaton's contribution evaluated the saving behavior of impatient, liquidity constrained agents. He showed that if the agent's income is stationary, i.i.d. distributed over time, the equilibrium path is characterized by a low average level of assets, which accomplishes significant smoothing. If the income is characterized by a positively autocorrelated process, the role of assets as buffer stock will decline the higher the autocorrelation is, and will practically disappear when the income process approaches a random walk.

[7] However, their results show that even their small fund could have had a considerable effect on expenditure volatility (reducing the standard deviation of expenditure by about 40%).

[8] The Chilean CSF operated since 1987 to dampen fluctuations in fiscal expenditure arising from fluctuation in copper prices (copper accounts for about 30% of Chile's export). It defines a reference price for copper linked directly to a 6 years moving average of the spot prices. If the realized copper price exceeds (falls short) of the reference price plus (minus) a threshold, the extra revenue is deposited (withdrawn) from the fund. This fund proved useful during turbulent periods such as the 'poisoned grapes' period where Chilean exports to the US were severely restricted and exporters were compensated using US$363 million drawn on the Chilean CSF account. For further discussion on rules for shock management in Latin America see Hausmann and Gavin (1995).

[9] For example, estimates of the fiscal impact of the oil shocks in Venezuela in the 1970s were of the order of 11% of the GDP, and of about 5% of the GDP in Chile in the 1980s.

[10] See Tornell and Lane (1994) and Aizenman and Powell (1995).

[11] See Basch and Engel (1993) and Hausmann et al. (1993).

tions that were not dealt with in this paper. The purpose of this paper is not to claim that CSF are always welfare enhancing. Instead, we illustrate that results that hinge upon the size of the risk premium in an expected utility maximization model should be reexamined in light of the generalized expected utility maximization literature. Various predictions of previous frameworks may be modified substantially, even before adding factors like adjustment costs and political economy considerations.

We close the paper with a reexamination of precautionary savings. As was shown by Leland (1968) and Sandmo (1970), higher volatility of future income has an ambiguous effect on precautionary saving, an effect the sign of which hinges on the concavity of the marginal utility. We point out that their results are significantly modified when agents are disappointment averse. While the concavity of the marginal utility has the same effect as in the above studies, its magnitude is of a *second order*, whereas disappointment aversion has a *first order* effect on precautionary saving. For a disappointment averse agent, higher volatility will increase precautionary saving in proportion to the standard deviation. Hence, for a disappointment averse agent higher volatility increases the demand for saving independently from the concavity of marginal utility. Section 2 reviews the concept of disappointment aversion agents. Section 3 applies it to characterize an optimal buffer stock for a disappointment averse agent. Section 4 evaluates the impact of disappointment aversion on precautionary saving, and Section 5 closes with concluding remarks.

2. Disappointment aversion utility

We start this section with a brief discussion of generalized expected utility approaches (which include the disappointment aversion utility), and conclude with an overview of the approach adopted in this paper.

The expected utility maximization model outlined in Savage (1954) has been proven to be a very useful model, yet it has confronted difficulties in explaining various 'anomalies' and modes of behavior that do not correspond well with Savage's assumptions. For example, the evidence reported by Kahneman and Tversky (1979) and Tversky and Kahneman (1991) identified the asymmetric treatment of losses vs. gains — experiment subjects attach much greater disutility to a marginal income loss than the utility gain attributed to an equal income gain. This 'loss aversion' is inconsistent with Savage's expected utility framework, where the marginal utility associated with a gain equals (up to second order terms) the marginal utility of a loss. Generalized expected utility theories attempt to explain these findings by assuming that the agent attaches different values to the marginal utility of a gain vs. a loss.

It is noteworthy that interest in generalized expected utility models goes beyond explaining the loss aversion. Savage's expected utility maximization framework, while very useful in many contexts, has difficulties in explaining some 'anomalies' and modes of behavior. For example, simultaneous gambling and insurance, the excess volatility of stock prices reported by Shiller (1981), and the equity risk-premium puzzle identified by Mehra and Prescott (1985) raise questions about the appropriateness of Savage's approach. The Allais paradox suggests that there are interesting situations where the bias towards a known outcome impacts decision-making in ways that are not captured in Savage's environment. These kinds of concerns have led to the development of generalized expected utility approaches that relax Savage's axioms. While the debate about the merits of these new approaches is not yet settled, the new approaches deserve further theoretical and empirical investigation. The empirical evidence inspired by the Allias paradox suggests that there are interesting situations where the presence of 'certainty bias' impacts decision making in ways that are not modeled well in Savage's environment. These concerns have led to the development of generalized expected utility approaches, relaxing Savege's axioms. Gul's disappointment aversion is an example of one possible extension.

The preferences of a disappointment aversion agent may be summarized by $[u(x),\ \beta]$, where u is a conventional utility function describing the utility of consuming x, $[u' > 0, u'' < 0]$, and $\beta \geq 0$ is a number that measures the degree of disappointment aversion.[12] The disappointment adverse expected utility is defined implicitly, by describing its key features. In the absence of risk, the agent's utility level is simply $u(x)$. Suppose that our consumer faces risky income $\{x_s\}$ in n states of nature, $s = 1, \ldots n$. Let us denote by $V(\beta;\{x_s\})$ the expected utility of a disappointment averse agent (whose disappointment aversion rate is β). Let μ denote the certain income that yields the same utility level as the risky income: $V(\beta;\{x_s\}) = u(\mu)$.[13] Our consumer is revealing disappointment aversion if he/she attaches extra disutility to circumstances where the realized income is below μ. A convenient way to define $V(\beta;\{x_s\})$ is

$$\begin{aligned} V(\beta;\{x_s\}) &= \int u(x)f(x)\mathrm{d}x - \beta\int_{\mu>x}[u(\mu)-u(x)]f(x)\mathrm{d}x \\ &= E(u(x)) - \beta E[u(\mu)-u(x)|\,\mu > x]\Pr[\mu > x], \end{aligned} \tag{1}$$

where f is the density function, E is the expectation operator, and $\Pr[z]$ is the probability of event z, and $E[u(\mu) - u(x)|\,\mu > x]$ is the expected value of $u(\mu) - u(x)$, conditional on the realized consumption being below the certainty equivalent consumption. The term $E[u(\mu) - u(x)|\,\mu > x]$ measures the average 'disappointment', being defined by the expected difference between the certainty equivalence utility and the actual utility u in states of nature where the realized income is below the certainty equivalence income. The disappointment averse expected utility equals the conventional expected utility, adjusted downwards by a measure of disappointment aversion (β) times the 'expected disappointment'.

[12] Gul (1991) considered the more general case, where u may be both convex and concave. We restrict our attention to $u'' < 0$.

[13] That is, the consumer is indifferent between the prospect of a safe income μ and risky income $\{x_s\}$ in n states of nature ($s = 1, \ldots n$).

Gul's specification should be viewed as a special example of models where agents have some reference level of consumption. They weigh losses relative to the reference level, differently from weighing gains relative to this reference point [see Kahneman and Tversky (1979) for a pioneering construction of such a model]. The various models differ in the choice of the reference level of consumption. Gul's model uses the certainty equivalent of the lottery as the reference level of consumption, whereas Kahneman and Tversky (1979) use today's consumption. It can be verified that the logic of our discussion does not depend on the precise choice of the reference point. The key assumption driving our result is the loss aversion — the notion that losses weigh more than gains relative to the given reference point [see Bowman et al. (1997) for a related paper that uses a different reference point in another context].

We restrict our attention first to the simplest example — of two states of nature. Suppose that the agent will receive income x_i in state i ($i = 1, 2$), where $x_1 > x_2$, with probabilities $(\alpha, 1-\alpha)$, respectively. Applying Eq. (1), the disappointment averse expected utility is defined by:

$$V(\beta) = \alpha u(x_1) + (1-\alpha)u(x_2) - \beta(1-\alpha)[V(\beta) - u(x_2)], \tag{2}$$

where for notation simplicity we henceforth use $V(\beta)$ for $V(\beta;\{x_s\})$. Thus,

$$V(\beta) = \frac{\alpha}{1+(1-\alpha)\beta}u(x_1) + \frac{(1-\alpha)(1+\beta)}{1+(1-\alpha)\beta}u(x_2). \tag{2'}$$

Note that for $\beta = 0$, V is identical to the conventional expected utility.[14] We turn now to identify the risk premium in the presence of disappointment aversion. Suppose that a disappointment averse agent, described above, faces income $[Y+\varepsilon, Y-\varepsilon]$, and let $\alpha = 0.5$. In these circumstances the expected utility of the agent is

$$V = \frac{0.5}{1+0.5\beta}[u(Y+\varepsilon) + (1+\beta)u(Y-\varepsilon)]. \tag{2''}$$

Eq. (2″) indicates that the ratio of the marginal utility of a loss to the marginal utility of a gain is $1+\beta$. The literature refers to this ratio as a measure of loss aversion, since it captures a person's relative sensitivities to losses vs. gains. This ratio is 1 in the conventional Savage framework, and exceeds 1 by the disappoint-

[14] An alternative way of writing the disappointment averse expected utility is

$$V(\beta) = \alpha\left[1 - \frac{(1-\alpha)\beta}{1+(1-\alpha)\beta}\right]u(x_1) + (1-\alpha)\left[1 + \frac{\alpha\beta}{1+(1-\alpha)\beta}\right]u(x_2).$$

If the agent is disappointment averse ($\beta > 0$), he attaches extra weight to 'bad' states where he would be disappointed (relative to the probability weight used in the conventional utility), and attaches a lighter weight to 'good' states.

ment aversion coefficient (β) in Gul's framework. Empirical estimates of loss aversion are typically in the neighborhood of 2, indicating that $\beta = 1$ if the agents' preferences conform to Gul's framework (see Kahneman et al., 1990; Tversky and Kahneman, 1991).

We can use Eq. (2″) to link disappointment aversion with risk premium. Define the risk premium τ associated with a symmetric gamble by:

$$u(Y - \tau) = \frac{0.5}{1 + 0.5\beta}u(Y + \varepsilon) + \frac{0.5(1 + \beta)}{1 + 0.5\beta}u(Y - \varepsilon).$$

Applying a second order Taylor approximation leads to

$$\frac{\tau}{Y} \approx \frac{0.5\beta}{1 + 0.5\beta}\sigma_y + 0.5R[\sigma_y]^2, \tag{3}$$

where R, σ_y are the coefficient of relative risk aversion and the coefficient of variation of income, respectively: $R = -Y(u''/u')$, $\sigma_y = \varepsilon/Y$. Note that the risk premium increases with the degree of disappointment aversion times the coefficient of variation. Furthermore, if $\beta > 0$ then the disappointment aversion may dominate the determination of the risk premium, as the relative risk aversion R is playing only a secondary role (i.e. the impact of R is proportional to the square of the coefficient of variation, whereas the impact of β is proportional to the coefficient of variation).[15] Hence, the addition of disappointment aversion may modify substantially all the results that hinge on calculations involving risk premium. To better appreciate this observation, we turn now to examine how disappointment aversion affects the demand for stabilization funds.

It is noteworthy that analogous results can be obtained if the agent would maximize a Savage type of utility with kinks. A shortcoming of Savage's approach is that these kinks are exogenously specified, whereas in Gul's specification the kink is endogenously determined by the proper reference point. This observation applies to other generalized expected utility approaches. In fact, if the reference point is determined by some kind of average past experience, one should be able to derive similar results in the context of habit formation models.

3. Optimal buffer stock and disappointment aversion

There exists a large literature dealing with optimal buffer stock and CSF rules, with varying degrees of complexity of assumptions regarding preferences, budget constraints, and stochastic processes. To illustrate our point we focus on one of the simpler examples advanced by Newbery and Stiglitz (1981, pp. 415–420; NS

[15] Obviously, this statement is conditional on the actual size of the coefficient of variation of shocks. To get some perspective, note that the coefficient of variation of output and of the real exchange rate of developing countries is typically below 0.15 (implying that $(\sigma_y)^2 < 0.0225$).

henceforth). This allows us to exemplify the implications of disappointment aversion on buffer stocks and international reserves in a rather tractable way. Similar points can be made in the context of more complex models.

Consider the case where there are equally likely two states of the world: high and low real output (h):

$$h = 1 \pm \sigma, \qquad \sigma \geq 0. \tag{4}$$

The consumer's utility function is

$$u(c) = \begin{cases} \dfrac{c^{1-R}}{1-R}, & R \neq 1, \quad R \geq 0 \\ \log c, & R = 1 \end{cases} \tag{5}$$

For simplicity of exposition, we ignore discounting and consider a simple storage rule: the goal is to stabilize consumption around the mean real income $[E(h) = 1]$. In good years agents put into storage σ, as long as the storage capacity has not been reached. In bad years agents take away from the storage σ, as long as the storage is not empty.[16] Suppose that the capacity of the storage is K, and to simplify assume that K is an integer, and define a 'storage unit' as σ units of output. Hence, there are $K + 1$ possible levels of stock $[0, 1, \ldots K$ units of storage$]$. Let π_V be the probability that the stock contains exactly V units. NS showed that in these circumstances[17]

$$\pi_V = \frac{1}{K+1}. \tag{6}$$

Hence, there is a uniform distribution of amounts of storage, and consumption is stabilized at level 1 in $K/[K+1]$ of the time. It is $1+\sigma$ in $0.5/[K+1]$ of the time in cases where the storage is full and there is a good state of nature, and $1-\sigma$ in $0.5/[K+1]$ of the time in cases where the storage is empty and there is a bad state of nature. Hence, the expected utility with buffer stock (in the absence of discounting) is

$$V_b = \frac{K}{K+1}u(1) + \frac{u(1-\sigma) + u(1+\sigma)}{2(K+1)}, \tag{7}$$

[16] This rule can be shown to be optimal if the storage cost is zero.

[17] This follows from the fact that if the stock at time t is V, it was either $V-1$ or $V+1$ at time $t-1$ with equal probabilities (unless $V = 0$ or $V = K$). Hence, $\pi_0 = 0.5[\pi_1 + \pi_0]$, $\pi_V = 0.5[\pi_{V+1} + \pi_{V-1}]$ for $1 \leq V < K-1$, and $\pi_K = 0.5[\pi_K + \pi_{K-1}]$. Solving these $K+1$ equations we infer Eq. (6). Note that the buffer stock is characterized by $K+1$ possible states. The above discussion provides an example of dealing with more than two states of nature in Gul's framework. In a similar way one can extend the discussion to deal with more than two states of nature.

where index b indicates an active buffer stock. In the absence of a buffer stock, the expected utility is

$$V_n(0) = \frac{u(1-\sigma) + u(1+\sigma)}{2}, \tag{8}$$

where index n indicates no active buffer stock. Applying a second order approximation the expected benefit from the buffer stock is proportional to the conventional risk premium:

$$B = \frac{V_b(0) - V_n(0)}{u'(1)} \approx \frac{R\sigma^2 K}{2(K+1)}. \tag{9}$$

Let the storage cost per period per unit of storage (inclusive of capital cost) be s, and let ξ be the cost of maintaining a storage capacity of a unit. The total expected cost of running the buffer stock (per period) is

$$[0.5s + \xi]K\sigma \tag{10}$$

The optimal buffer stock capacity K^* is obtained by maximizing the expected net cost of operating the buffer stock:

$$\operatorname*{Max}_K \{B - [0.5s + \xi]K\sigma\} \tag{11}$$

leading to

$$K^* \approx \sqrt{\frac{R\sigma}{s + 2\xi}} - 1. \tag{12}$$

For example, if $R = 1$, $\sigma = 0.5$, $s + 2\xi = 0.1$ the optimal storage is $K^* = 1$, the net benefit from the storage is 3.75%, and income is stabilized in half of the time (these numbers were used by NS).

These results have led NS to conclude that 'even for large risk and low storage costs the average buffer stock is small, and that the buffer rule which would be optimal if costs are ignored often looks relatively unattractive compared with alternatives once the costs are included'.[18]

We turn now to a recalculation of the optimal storage for the case where agents are disappointment averse. Specifically, suppose that our agent is characterized by the utility specified in Eq. (1). This does not change the operation of the buffer

[18] One can apply NS methodology to get similar welfare inferences using Deaton's results. For example, if the income process is i.i.d. with a standard deviation of 10%, the optimal application of precautionary saving by an impatient, liquidity constrained agent reduces the consumption standard deviation to 5% for a consumer whose coefficient of risk aversion is $R = 2$ (see Deaton, 1991; p. 1234). The resultant consumption smoothing can be translated into an income gain of about 0.75% (i.e. the consumer is willing to sacrifice up to 0.75% of his average income for the option to smooth consumption optimally).

stock, as characterized by Eq. (6), but it modifies the evaluation of the expected benefit. The disappointment averse expected utility in the absence of stabilization is:

$$V_n(\beta) = \frac{0.5}{1 + 0.5\beta}[u(1 + \sigma) + (1 + \beta)u(1 - \sigma)]. \tag{13}$$

Applying Eq. (1) the disappointment aversion expected utility (with active stabilization) is[19]

$$V_b(\beta) = \frac{K}{K + 1}u(1) + \frac{u(1 - \sigma) + u(1 + \sigma)}{2(K + 1)} - \beta\frac{0.5}{K + 1}\{V_b(\beta) - u(1 - \sigma)\}. \tag{14}$$

Alternatively,

$$V_b(\beta) = \frac{Ku(1) + 0.5u(1 + \sigma) + (1 + \beta)0.5u(1 - \sigma)}{K + 1 + 0.5\beta}. \tag{15}$$

Applying a second order Taylor expression we obtain that

$$B(\beta) = \frac{V_b(\beta) - V_n(\beta)}{u'(1)} \approx \frac{K}{K + 1 + 0.5\beta}\left[0.5R\sigma^2 + \frac{0.5\beta}{1 + 0.5\beta}\sigma\right]. \tag{16}$$

A comparison of Eq. (3) and Eq. (16) reveals that the benefit from the buffer stock is proportional to the modified risk premium, which in turn depends linearly on the degree of disappointment aversion times the standard deviation of the shock. The optimal buffer stock is obtained by:

$$\underset{K}{\mathrm{Max}}\{B - [0.5s + \xi]K\sigma\} \tag{17}$$

leading to

$$K^* \approx \sqrt{\frac{(1 + 0.5\beta)R\sigma + \beta}{s + 2\xi}} - (1 + 0.5\beta). \tag{18}$$

[19] Eq. (14) evaluates the ‘asymptotic’ expected utility of a disappointment averse agent whose horizon is long. This is akin to the situation where the agent is randomly placed at a moment in time. Formally, one may consider the case of an agent whose horizon is t periods, $t = 1, 2\ldots$ The agent evaluates his expected utility per period by applying an intertemporal version of Eq. (1) for the case where μ is the certainty equivalent consumption throughout the *entire* t periods. The Markov chain properties of the buffer stock can be shown to imply that, while the value of the expected utility is conditional on the initial value of the buffer stock, it converges to Eq. (14) for a long enough time horizon (i.e. $V_b(\beta)|_t \underset{t\to\infty}{\rightarrow} V_b$ where $V_b(\beta)|_t$ is the expected utility of a consumer whose horizon is t).

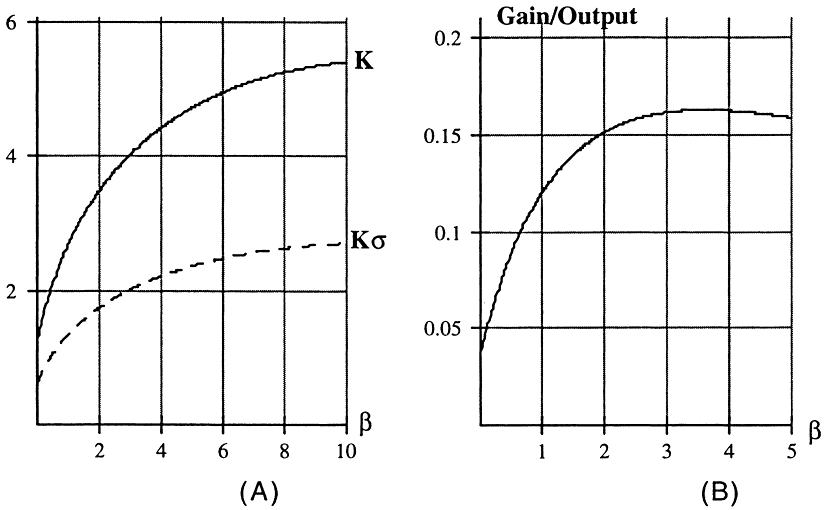

Fig. 1. Disappointment aversion buffer stocks (drawn for $\sigma = 0.5$, $R = 1$, and $s + 2\xi = 0.1$).

Fig. 1A summarizes the dependency of the buffer stock (K) and the average buffer stock $(K\sigma)$ capacity on the disappointment aversion. The NS results are obtained for $\beta = 0$ (the parameters in the simulation are the ones that NS used in their base case). Note that disappointment aversion has first order effects on the buffer stock. For example, if $\beta = 1$, then the optimal storage capacity is $K = 3$ (keeping in mind the integer restriction).[20] In this case we would observe that the average buffer stock is a year and a half output, and that consumption is stabilized in $3/4$ the time (whereas it is stabilized only in half of the time if $\beta = 0$, as was the case considered by NS). Fig. 1B plots the gain from the buffer stock, as a fraction of the annual output, revealing a gain of 12% of the GDP if $\beta = 1$ (and only 3.75% if $\beta = 0$). Both results reveal that disappointment aversion induces first order effects, changing significantly the predictions of the model.

In general, it is both the degree of disappointment aversion (measured by β) and the curvature of u (measured by R) that determine the ultimate use and the gains from buffer stocks. While the two aversions interact, the disappointment aversion has a robust role that is independent from the role of risk aversion. To better appreciate this point, we focus in Fig. 2 on the case where the coefficient of risk aversion is rather small ($R = 0.25$). Note that in the absence of disappointment aversion, the use of buffer stocks and their welfare implications are minimal. This changes drastically with disappointment aversion, which induces significant welfare gains from using a sizable buffer stock. Thus, from the two aversions, it is the

[20] Recall that $\beta = 1$ is consistent with estimates of the loss aversion. If $\beta = 2$, then the optimal storage capacity is $K = 4$, the average buffer stock is a 2 years output, and the consumption is stabilized in $4/5$ the time.

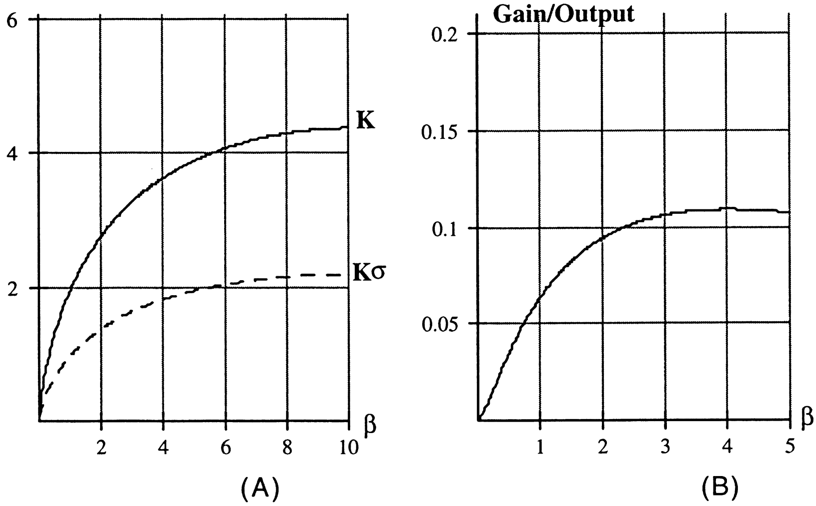

Fig. 2. Disappointment aversion buffer stocks (drawn for $\sigma = 0.5$, $R = 0.25$, and $s + 2\xi = 0.1$).

disappointment aversion that has the more decisive role in explaining the demand for buffer stocks.

The potentially large gains from managing buffer stocks raise the question why we do not see more aggressive use of such stocks by developing countries. The answer to this puzzle may rely on the fact that our paper focused on the behavior of a representative agent, ignoring political economy factors. These factors may limit the applicability of large buffer stocks, as such stocks will tend to be abused by policy makers whose horizon is short [see Tornell and Lane (1994) and Aizenman and Powell (1995) for further discussion]. The logic of our discussion is applicable beyond the buffer stock issue. It suggests that welfare calculations and demand for assets that depend on risk premiums should be adjusted if the agent is disappointment averse. The needed adjustment is proportional to the disappointment aversion times the standard deviation. This adjustment may change significantly previous results, as its impact is of a first order magnitude (compared to a second order magnitude of the conventional risk premium). We close our discussion with another illustration of this point, identifying the impact of volatility on precautionary saving.

4. Disappointment aversion and precautionary saving

Consider the problem of an agent who lives two periods. His first and second period budget constraints are

$$c_1 = x_1 - S$$

$$c_2 = x_2 + S(1 + r), \tag{19}$$

where c_i, x_i denote the consumption and endowment in period i ($i = 1, 2$), S is the saving, and r is the real interest rate. The agent determines saving by maximizing an intertemporal time additive utility:

$$U = u(c_1) + \frac{V_2(\beta)}{1 + \rho}, \tag{20}$$

where $V_2(\beta)$ is the disappointment averse second period utility, defined by Eq. (1) and Eq. (2). Suppose that the second period income rate may be either high or low, with equal probabilities:

$$x_2 = \begin{cases} \bar{x} + \sigma & \text{prob. } 0.5 \\ \bar{x} - \sigma & \text{prob. } 0.5 \end{cases} \tag{21}$$

for notational convenience we assume that the real interest rate equals the rate of time preferences. Applying Eq. (2″) the generalized expected utility is:

$$U = u(x_1 - S) + 0.5\frac{u[\bar{x} + S(1 + \rho) + \sigma] + (1 + \beta)u[\bar{x} + S(1 + \rho) - \sigma]}{(1 + \rho)(1 + 0.5\beta)}. \tag{22}$$

The first order condition characterizing saving is

$$u'(x_1 - S) = 0.5\frac{u'[\bar{x} + S(1 + \rho) + \sigma] + (1 + \beta)u'[\bar{x} + S(1 + \rho) - \sigma]}{1 + 0.5\beta}. \tag{23}$$

We identify the impact of volatility by a second order expansion of Eq. (23) around $\sigma = 0$. It can be shown that

$$\mathrm{d}S = \frac{\dfrac{0.5\beta}{1 + 0.5\beta}\sigma + 0.5\left[\dfrac{u'''[\bar{x} + S(1 + \rho)]}{u'[\bar{x} + S(1 + \rho) + \sigma]}\right]\sigma^2}{A_1 + (1 + r)A_2} \tag{24}$$

here A_i is the coefficient of absolute risk aversion at time i (i.e. $A_i = -u_i''/u_i > 0$).

In the absence of disappointment aversion, Eq. (24) states the results advanced by Leland (1968) and Sandmo (1970) — the impact of volatility on precautionary saving is determined by the concavity of the marginal utility of consumption, and its sign is ambiguous for a general utility. If the agent is disappointment averse, however, the concavity of the marginal utility is playing only a secondary role relative to the role of the disappointment aversion (as the first term is proportional to the coefficient of variation, and the second to the coefficient of variation's square). Thus, volatility tends to induce a first order positive effect on precaution-

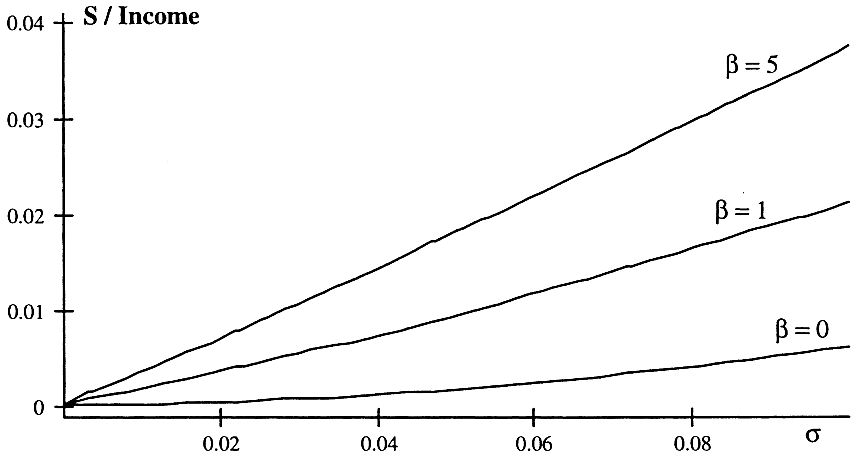

Fig. 3. Disappointment aversion and the precautionary saving (plotted for $\rho = 0.05$, $R = 1.5$).

ary saving independently of the concavity pattern of the marginal utility.[21] Fig. 3 plots a simulation of precautionary saving as a function of the coefficient of variation for the case where $R = 1.5$. Note that in these circumstances disappointment aversion magnifies precautionary saving, which is of secondary importance in the absence of disappointment aversion. Fig. 4 illustrates this point for a quadratic utility function.[22] While the precautionary demand for a quadratic utility is zero in the absence of disappointment aversion, it is of a first order magnitude in the presence of disappointment aversion.

5. Concluding remarks

In concluding the paper it is useful to put it in the context of recent literature. Savage's expected utility maximization framework, while very useful in many

[21] It is noteworthy that this result applies even if the income process follows a random walk. The rationale is that with a random walk process the future income is distributed symmetrically around the present income. In these circumstances a disappointment averse agent will have a much greater incentive to hold assets as buffer stock compared to a disappointment neutral agent. The first treats the future asymmetrically, assigning a greater weight to bad states of nature. A greater coefficient of disappointment aversion increases the gap between the weights attached to the bad vs. the good states, increasing thereby the gains from the buffer stock. Hence, even if the income process approaches a random walk, a disappointment averse agent still has a powerful incentive to save. In contrast, a disappointment neutral agent treats the future states of nature symmetrically. The only reason to save stems from the curvature of the marginal utility, which generates only a second order magnitude of saving.

[22] Figs. 3 and 4 report the results using the exact functions, and not the Taylor approximation. Note that our normalization of the mean to 1 implies that σ is also the coefficient of variation. Extending the simulation to larger values of σ would only magnify the results, without changing the main insight.

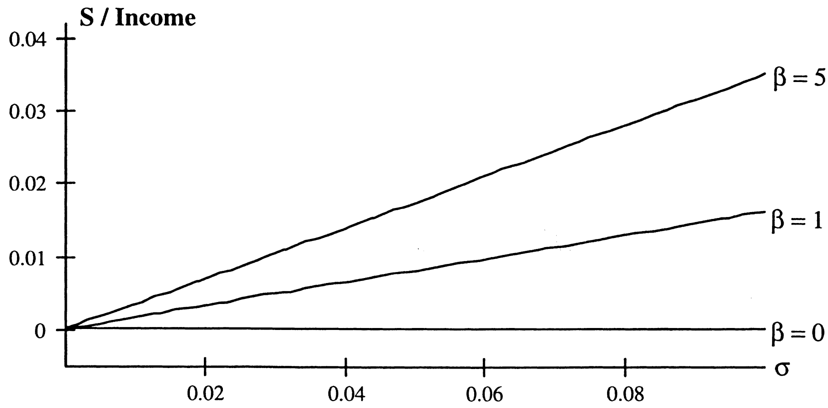

Fig. 4. Precautionary saving, disappointment aversion and quadratic utility [drawn for $u'(x) = 10 - 1x$].

contexts, has difficulties in explaining some 'anomalies' and modes of behavior. These concerns have led to the development of generalized expected utility approaches that relax Savage's axioms. Disappointment aversion, while convenient, is only one of the many extensions of expected utility [see Epstein (1992) for a useful review of the new approaches to modeling risk]. Some of the other extensions have been already applied in Macroeconomics. For example, Schmeidler (1989) and Gilboa (1987) modeled a nonadditive subjective probability framework to account for rational decision making under Knightian uncertainty.[23] Other generalizations of expected utility include recursive utility, where the timing of risk resolution matters, along the lines of Kreps and Porteus (1978).[24] While the debate regarding the merits of generalized expected utility is not over, these studies point out that the choice of the framework has important bearings on practical policy questions.

Loss aversion and generalized expected utility models induce one to change the outlook on most issues involving uncertainty and risk. This paper focused on buffer stocks, but its message has direct bearings on the welfare benefits of using precautionary saving to smooth consumption. It suggests that using the conventional expected utility model leads one to understate the costs of suboptimal savings patterns. Applications of generalized expected utility may provide a rationale for the observed large adverse effects of economic volatility on economic growth and

[23]This approach was applied by Epstein and Wang (1994) and Dow and Ribeiro de Costa (1992), who showed that excess volatility of the type reported by Shiller is consistent with equilibrium outcomes under Knightian Uncertainty, and by Aizenman (1997) who showed that uncertainty may inhibit growth and may lead to first order costs.

[24]Versions of this approach were applied by Epstein (1991) to explain macro consumption and asset price data. In another context, Epstein and Zin (1990) showed that allowing for 'first-order' risk aversion helps considerably in accounting for the observed equity premium, by generating both a small risk free interest rate and a moderate equity premium.

private investment (see Ramey and Ramey, 1995; Aizenman and Marion, 1998). It may provide an interpretation for puzzling patterns of portfolio investment, like the equity premium puzzle, the 'home bias' in the allocation of international portfolios, and the excess volatility of prices (see Epstein and Wang, 1994). While none of these extensions is free from challenges, it represents an exciting research direction that deserves further theoretical and empirical underpinning.

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