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Optimal tax and debt policy with endogenously imperfect creditworthiness

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Abstract

This paper studies the patterns of optimal tax rates and borrowing in a developing country characterized by a costly tax collection. Its access to the international credit market is determined by the efficiency of the tax system, the relative bargaining power of creditors, and the outstanding debt. Country risk modifies considerably the pattern of taxes and borrowing in recessions. The tax rate exhibits strong counter-cyclical patterns in economies operating close to the credit ceiling, whereas the tax rate exhibits very few cyclical patterns in economies operating on the elastic portion of the supply of credit, where country risk factors are absent.

Keywords

Borrowing constraints, credit ceilings, optimal tax

1. INTRODUCTION

In recent years, the theory of public debt management has made important strides. Barro (1979) provided the foundation for a neoclassical theory of debt management with testable implications for the management of public debt, under the assumption that governments behave in the manner that theory suggests would be optimal. And in fact, for industrial economies, the evidence is at least roughly consistent with the predictions of the theory.[1]

However, the theory appears to have rather less explanatory power for developing economies. In particular, a key implication of the theory is that the public deficit should be counter-cyclical, rising when income (or, more generally, the tax base) is below its long-run trend and declining when income is transitorily high. Although this prediction is borne out for industrial economies, Gavin *et al*. (1996) show that deficits are much less counter-

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cyclical in Latin America than in the industrial economies.[2] Indeed, during periods of low economic growth, the Latin American deficit actually moves in the 'wrong' direction, from the perspective of both the neoclassical and Keynesian theories of optimal fiscal policy. Table 1 illustrates this by summarizing the relationship between output growth and fiscal outcomes in Latin America and the industrial economies during major recessions.

In the industrial economies, the typical 'major' recession involved a decline in real GDP of 3.3 percent, and coincided with a reduction in the overall fiscal surplus of about 3.0 percent. In Latin America, the typical major recession involved a decline in real GDP of nearly 11 percent, and a movement into fiscal surplus of nearly 2 percentage points of GDP.

The evolution of fiscal policy in Argentina and Mexico during 1995 provides a striking and revealing illustration of this pro-cyclical fiscal response. In both countries, concerns about the policy environment in the aftermath of Mexico's December 1994 devaluation led to large capital outflows in the first months of 1995. Despite an international rescue effort of unprecedented scale, sentiment about both countries continued to deteriorate in the first two months of 1995, and what can be described as a full-blown financial panic continued to worsen until early March, when both the Argentine and the Mexican governments announced major economic adjustment packages. The core of each package was a major fiscal contraction, involving large tax increases and sharp cuts in real public spending. This fiscal contraction appears to have succeeded in calming the financial panic, thus setting the stage for eventual economic recovery. However, coinciding as it did with deep recessions in both countries, the fiscal contraction comprised a highly pro-cyclical fiscal reaction that stands in sharp contrast to the more counter-cyclical response normally observed in industrial economies.[3]

Why is it that the conventional theory appears to explain fiscal outcomes in the industrial economies, but not in Latin America? There are two potential explanations; either the assumptions of the theory provide a poor approximation of the economic environment in which Latin American decision makers must act, or policy makers in the region have been less able to set fiscal policy in conformance with the dictates of economic efficiency. While neither explanation can be ruled out, in this paper we explore the first, focusing on the implications for optimal tax and debt policy of the possibility of public default.[4] Unlike in industrial countries, where the risk of default is low, governments in many developing countries have a relatively recent history of default on both domestic and external debt, and the possibility of such default is reflected in high and variable risk premia on developing countries' foreign-currency denominated debt. It is a fact that many developing countries have limited and sporadic access to international credit markets; in this paper, we attempt to explain this fact, and argue that it has important implications for optimal tax and public debt policy.

We construct a model in which default can occur and, in response to this

Table 1 Fiscal response to major recessions – Latin America and industrial economies. Consoliated fiscal government

	Industrial economies			Latin America		
	Cumulative changes in real GDP (%)	Changes in total surplus, (% of GDP)	Change in primary surplus (% of GDP)	Cumulative change in real GDP (%)	Changes in total surplus (% of GDP)	Change in primary surplus (% of GDP)
Average change	–3.3	–3.0	–2.3	–10.7	1.6	1.8
Standard deviation	2.6	3.6	3.3	7.2	4.5	4.3

Source: Gavin and Perotti (1996). Major recessions are defined as periods during which the cumulative decline in real GDP is larger than 4.0 percent (Latin America) and 1.5 percent (industrial economies). There were 26 such recessions in Latin America and 22 in the OECD.

All fiscal data are from the IMF *Government Finance Statistics*. The consolidated central government was chosen as the unit of analysis for reasons of data availability, and because it comprises a political unit that can sensibly be viewed as making decisions on fiscal policy. Adequate data could be compiled for 13 Latin American economies, including Argentina, Bolivia, Brazil, Chile, Columbia, Costa Rica, the Dominican Republic, Mexico, Panama, Paraguay, Peru, Uruguay, and Venezuela. The industrial-economy sample comprises 21 countries Australia, Belgium, Canada, Denmark. Finland, France, Germany, Great Britain, Iceland, Ireland, Italy, Greece, Japan, the Netherlands, Portugal, Spain, Sweden, Switzerland, and the United States.

fact, borrowing constraints endogenously arise. The possibility of default and the resulting borrowing constraints arise from the interaction of inefficient tax systems and a volatile tax base. For countries with efficient tax systems, a relatively stable tax base and a low level of inherited debt, the probability of default is negligible and, as in Barro (1979), public borrowing is utilized to stabilize tax rates in the face of transitory shocks to the tax base. Recessions have little or no effect on tax rates, and deficits are counter-cyclical.

When the domestic tax system is inefficient and the tax base volatile, a sufficiently large recession will force the government to its credit ceiling, resulting in a complete loss of market access, in which case the government will be forced to increase tax rates when output and the tax base decline. Intertemporal tax-rate smoothing no longer applies.

The gross deviation from tax-smoothing behaviour that occurs when governments lose all access to financial markets is neither subtle nor surprising. More interesting is the impact on optimal tax and public borrowing of an intermediate regime, in which markets are aware that the government may default in the second period, but the probability of default is sufficiently low to support public borrowing at a contractual lending rate that compensates investors for the default risk. We show that in this ‘imperfectly creditworthy’ regime, fiscal authorities have incentives to lower taxes and increase borrowing, thus increasing the probability of a default in the second period, even though the *ex ante* real rate of interest is unaffected by the possibility of default. This raises the interesting possibility that the observed tendency of many developing countries to increase their indebtedness to levels that seriously compromise their creditworthiness may be a logical consequence of their economic structure, specifically tax capacities that are limited relative to the volatility of the domestic tax base. We also show that, in the ‘imperfectly creditworthy’ regime, small declines in output may be met with substantial declines in the tax rate, and large increases in public borrowing, and vice versa. The relationship between transitory shocks to the tax base and optimal taxes and borrowing is thus highly nonlinear, and depends upon the government’s creditworthiness. When creditworthiness is complete, intertemporal tax-rate smoothing applies and public borrowing is counter-cyclical. When creditworthiness is limited, the tax rate declines when the tax base declines, and public borrowing is therefore even more counter-cyclical. When creditworthiness has been lost, the government loses its ability to finance higher deficits, and the tax rate becomes a decreasing, rather than an increasing, function of the tax base.

We also show that optimal borrowing depends in a complex way upon the volatility of the domestic tax base. When the initial level of debt is low, and therefore creditworthiness complete, an increase in the volatility of the domestic tax base lowers the level of borrowing. However, when volatility increases enough, the government is placed in the regime of partial creditworthiness, and the incentive to reduce current taxes and raise borrowing increases.

The paper is organized as follows. Section 2 reviews the model. Section 3 identifies the optimality conditions that determine the intertemporal distribution of taxes and public borrowing. Section 4 investigates the optimal response of tax rates and public borrowing to transitory shocks to the tax base, identifies the effects of a switch from the no-default to the imperfectly-creditworthy regime, and examines the welfare effects of changing creditors' bargaining power. Section 5 studies the impact of volatility on optimal public borrowing, and Section 6 concludes.

2. THE FRAMEWORK

We consider a developing country characterized by a costly tax collection system, and limited access to the international capital market.

2.1 Taxes and collection costs

Taxes are costly, due to distortions and collection costs. The country's access to the international credit market is restricted by its ability to raise taxes. This would be the case if most borrowing is done by the public sector (including the case where the government 'insures' private borrowing).[5] To simplify, we focus on a two period example, where the output is random. The GDP in period i ($i = 1, 2$) is

$$Y_i = Y_i^0 + \varepsilon_i \tag{1}$$

where Y_i^0 is the output in the absence of productivity shocks, and ε_i is the productivity shock the support of which is

$$-\delta_i \leq \varepsilon_i \leq + \delta_i \quad \text{for } 0 \leq \delta \tag{2}$$

Let $f(\varepsilon)$ be the density function of productivity shocks.

A tax at rate t yields a net tax revenue of:

$$T_i(t) = Y_i[t - \Gamma(t)]; \quad \Gamma' \geq 0, \Gamma'' \geq 0 \tag{3}$$

The term Γ measures the loss associated with tax collection, which is assumed to be a convex function of the tax rate. We denote by t^*, T^* the maximum tax rate and the net tax revenue, defined implicitly by

$$1 = \Gamma'(t^*) \tag{4}$$

$$T_i^* = Y_i[t^* - \Gamma(t^*)] \tag{5}$$

We refer to T^* as the tax capacity.[6]

For example, if the tax collection cost function is quadratic:

$$T_i = Y_i \left[t - \frac{\lambda}{2} t^2 \right]; \qquad \lambda \geq 0 \tag{3'}$$

The maximum tax rate and the tax capacity are

$$\left. \begin{array}{ll} t^* = \dfrac{1}{\lambda},\ T_i^* = \dfrac{Y_i}{2\lambda} & \text{if} \quad \lambda \geq 1 \\ t^* = 1,\ T_i^* = Y_i \left[1 - \dfrac{\lambda}{2} \right] & \text{if} \quad \lambda < 1 \end{array} \right\} \tag{4'}$$

where λ measures the relative inefficiency of the tax system.

2.2 Borrowing constraints

The tax revenue is used to purchase public goods G (such as defence, health, etc). Public goods G are exogenously given, and have first priority before external debtors. The country may borrow internationally, but its borrowing ability is restricted by its tax capacity and the enforceability of international contracts, similarly to Helpman (1989). To simplify the analysis we consider the case where there is no domestic borrowing, and the initial outstanding foreign debt is zero (Appendix A reviews the case where the initial foreign debt is positive). Let B_1 stand for borrowing in period 1, at a contractual interest rate of r, and let r^* denote the risk free interest. Let S_2 stand for the debt repayment to foreign creditors in period 2. If the borrowing country will default in the second period, creditors would be able to 'confiscate' up to $\alpha[T_2^* - G]$, where $T^* - G$ stands for the *net* tax capacity (defined by the tax capacity T^* minus G, the government expenditure). The parameter α reflects the bargaining power of foreign lenders, indicating that a fraction α of the net tax capacity can be 'confiscated' due to the threat of embargoes, etc.[7] Consequently, The effective ceiling on net resource transfers to creditors is

$$S_2 = min[(1 + r)B_1; \quad \alpha[T_2^* - G]] \tag{6}$$

The international credit market is risk neutral, characterized by competition among banks that are fully informed regarding the debt exposure of the country. Hence, the interest rate r is determined by the condition that the expected yield on the debt equals the risk free interest rate

$$B_1(1 + r^*) = E\{S_2\} \tag{7}$$

where E is the expectation operator. Hence, if the repayment ceiling is not binding in all future states of nature, sovereign risk is absent, and competition equates the interest rate with the risk free interest rate ($r = r^*$). If the repayment ceiling is binding in some future states of nature, we observe a risk premium ($r > r^*$), determined by condition (7).

Note that (6) and (7) imply that $B_1(1 + r^*) = E\{S_2\} \leq E\{\alpha[T_2^* - G]\}$. Consequently, the borrowing ceiling on B_1 is given by $B_1 \leq \dfrac{E\left\{\alpha\left[T_2^* - G\right]\right\}}{1 + r^*}$.

A lower tax capacity (T^*), a higher fiscal commitment (G) or a lower creditor bargaining strength (α) induce a drop in the credit ceiling. Appendix A generalizes (7) to the case of a positive outstanding debt ($D_1 > 0$). For simplicity of exposition, the paper focuses on the case where $D_1 = 0$, but all its results are applicable for the general case.

2.3 The supply of credit

Equation (7) defines implicitly the supply of credit. Let $Y_1^0 = Y_2^0 = Y_0$, and assume the quadratic tax function (3′), with $\lambda \geq 1$. The debt $(1 + r)B_1$ will be repaid fully in period 2 if it falls short of the repayment ceiling, $\alpha[T_2^* - G]$. Let us denote by ε^* the productivity shock associated with the switch to the partial default regime – it is the highest second period output shock associated with partial default, defined by

$$(1 + r)\, B_1 = \alpha\left[\frac{Y_0 + \varepsilon^*}{2\lambda} - G\right] \tag{8}$$

If the probability of partial default in period 2 is zero (as is the case if $(1 + r^*)\, B_1 < \alpha\left[\dfrac{Y_0 - \delta_2}{2\lambda} - G\right]$, ε^* is set to equal $-\delta_2$. The interest rate facing the country, r, is determined by

$$B_1(1 + r^*) = \int_{-\delta_2}^{\delta_2} S_2(\varepsilon)\, f(\varepsilon)\mathrm{d}\varepsilon = \int_{-\delta_2}^{\varepsilon^*} \alpha\left[\frac{Y_0 + \varepsilon}{2\lambda} - G\right] f(\varepsilon)\mathrm{d}\varepsilon + \int_{\varepsilon^*}^{\delta_2} (1 + r)\, B_1\, f(\varepsilon)\mathrm{d}\varepsilon \tag{9}$$

Equation (9) defines implicitly the supply of credit facing the economy:

$$r = r[B_1;\quad \alpha, G, \lambda] \tag{9′}$$

Applying (9) it follows that

$$\frac{d\{B_1(1+r)\}}{dB_1} = \frac{1+r^*}{\int_{\varepsilon^*}^{\delta_2} f(\varepsilon)\,d\varepsilon} \tag{9''}$$

and for credit associated with a positive risk premium $\frac{dr}{d\alpha} < 0; \quad \frac{dr}{d\lambda} > 0;$. $\frac{dr}{dG} > 0.$

For example, if the second period output shock follows a uniform distribution with support [$-\delta, +\delta$] , (9) implies a supply of credit given by

$$1+r = \begin{cases} 1+r^* & \text{if } (1+r^*)B_1 \le \alpha\left[\frac{Y_0-\delta}{2\lambda} - G\right] \\ \frac{\alpha}{B_1}\left[\frac{Y_0+\delta}{2\lambda} - G\right] - \frac{\alpha\delta}{\lambda B_1}\sqrt{\frac{2\lambda}{\delta}\left[\frac{Y_0}{2\lambda} - G\right] - \frac{2\lambda B_1(1+r^*)}{\alpha\delta}} & \text{if } (1+r^*)B_1 > \alpha\left[\frac{Y_0-\delta}{2\lambda} - G\right] \end{cases} \tag{10}$$

Figure 1 summarizes the organization of the capital market. Panel I depicts the supply of credit facing the country, SS′S″. A country that operates on the flat segment SS′ exhibits full integration with the international capital market, and the absence of a risk premium. A country that operates along the upward sloping portion S′S″ exhibits partial integration with the capital market, and a rising risk premium. For each point along S′S″, the effective *ex post* interest rate [S_2/B_1] depends on the realized second period output. Panel II reports this dependency for point E [corresponding to $B_1 = 0.1$,panel I]. For low output, the *ex post* interest rate is state contingent, but is capped at $1 + r$ for high enough output. Our analysis will explore the implications of these nonlinearities on the behaviour of borrowing and tax rates throughout the business cycle.

There are two useful proxies for the debt exposure of a country. The first is the debt level associated with partial default [point S′], the second is the debt level associated with the credit ceiling [point S″]. For the case of a symmetric distribution of productivity shocks, and an expected output that is normalized to 1, the two measures as a fraction of the GDP are $\alpha\left[\frac{1-\delta}{2\lambda} - g\right]; \alpha\left[\frac{1}{2\lambda} - g\right]$, where g equals the GDP fraction of G. The debt service of a country as a fraction of the GDP is large if it approaches these measures. Countries with the same debt per capita are riskier if they are exposed to greater volatility, have less efficient tax systems, have high fiscal commitments, and low enforceability of international debt contracts (due to low openness, political instability, etc).

3. OPTIMAL BORROWING AND TAXES

The problem of the policy maker at time 1 is to determine the borrowing/saving ratio that will maximize the expected utility, subject to all the relevant budget constraints. The net tax revenue needs are determined according to:

$$T_1 = G - B_1$$

and (11)

$$T_2 = S_2 + G$$

We denote by ξ the fiscal demand for net tax revenue/GDP ratio

$$\xi_1 = \frac{G - B_1}{Y_1}; \ \xi_2 = \frac{S_2 + G}{Y_2}.$$

Applying (3) and (11) we solve for the tax rate as a function of ξ

$$t_1 = t(\xi_1); \quad t_2 = \mathrm{t}(\xi_2) \quad t' > 0 \tag{12}$$

The quadratic specification (3′) can be shown to imply that for

$\xi_i \leq \frac{1}{2\lambda}$ and $1 \leq \lambda$,

$$t_i = \frac{1 - \sqrt{1 - 2\lambda\xi_i}}{\lambda} \tag{13}$$

3.1 Optimal borrowing with endogenous partial defaults

The optimal borrowing B_1 attempts to 'smooth' consumption subject to the budget constraints and the given tax system. The preferences are a conventional time separable utility:

$$U = u(C_1) + \frac{1}{1 + \rho} u(C_2) \tag{14}$$

The consumption in each period equals the GNP net of the 'gross' tax revenue (inclusive of the collection cost Γ)

$$C_i = Y_i[1 - \xi_i - \Gamma(t(\xi_i))]; \quad i = 1,2\ ^{8} \tag{15}$$

Optimal borrowing is determined by:

$$\underset{B_1}{Max}\left[u(Y_1\{1-\xi_1-\Gamma(t(\xi_1))\})+\frac{1}{1+\rho}\int_{-\delta_2}^{\delta_2} u\left(Y_2\left[1-\frac{S_2+G}{Y_2+\varepsilon}-\Gamma\left(t\left(\frac{S_2+G}{Y_2+\varepsilon}\right)\right)\right]\right)f(\varepsilon)\mathrm{d}\varepsilon\right] \quad (16)$$

where S_2 is given by (7)–(9). Solving (16) we infer the first-order condition (see Appendix A for derivation):

$$u'(c_1)\ \{1+\Gamma'(\xi_1)\}=\frac{1+r^*}{1+\rho}E\ \{u\ (c_2)\{1+\Gamma\ (\xi_2)\}\}|\ \varepsilon>\varepsilon^*\} \quad (17)$$

where

$$\Gamma'(\xi)=\frac{\mathrm{d}\Gamma}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}\xi}$$

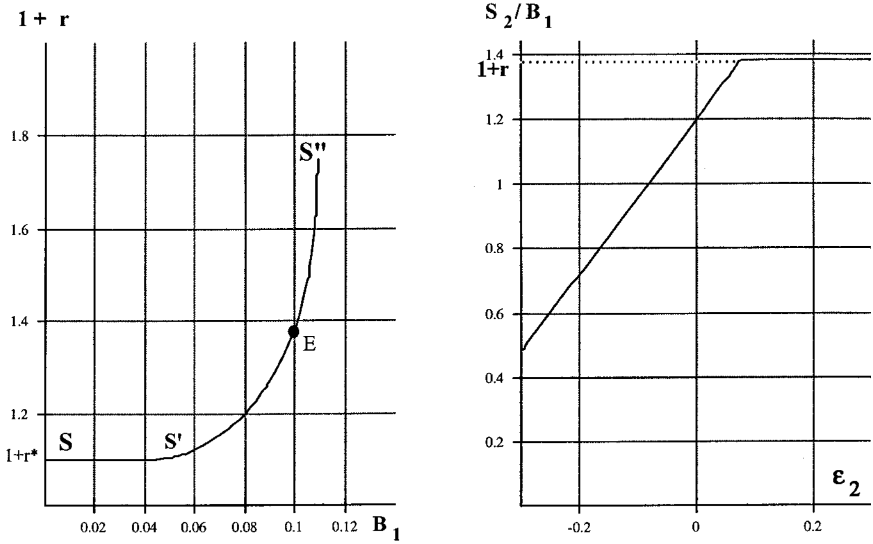
where $E[x|y]$ stands for the expected value of x, conditional on y. Optimal borrowing accomplishes distortion smoothing between period 1 and the states of nature in period 2 where *full repayment* occurs. Hence, shocks will impact borrowing and taxes by changing the intertemporal path of income, as well as by modifying the range of partial default.

The first-order condition in the absence of partial default is a special case of (17), where $\varepsilon^* = -\delta_2$:

$$u'(c_1)\ \{1+\Gamma'(\xi_1)\}=\frac{1+r^*}{1+\rho}E\ \{u'(c_2)\{1+\Gamma'(\xi_2)\}\} \quad (18)$$

Optimal borrowing in the absence of partial default equates the cost of public funds across time. The left-hand side (LHS) of (18) is the utility gain in period 1 associated with funding one dollar of fiscal expenditure by borrowing instead of taxing. The right-hand side is the expected utility cost of raising the future taxes needed to repay the first period borrowing.[9] If the consumer is risk neutral, and if $r^* = \rho$, optimal borrowing allows intertemporal smoothing of the tax burden, as in Barro (1979). In these circumstances the marginal cost of raising one dollar of net tax revenue at period 1 $[\Gamma'(\xi_1)]$ is equated with the expected cost of raising one dollar net taxes in the future $[E[\Gamma'(\xi_2)]]$. If consumers are risk averse, the above condition is modified – the monetary units are evaluated using the corresponding marginal utilities.

The main difference between the conventional analysis of optimal tax and our framework is the introduction of country risk considerations, implying that borrowing is done along a non-monotonic supply of credit facing the economy (as depicted in Figure 1). In these circumstances, if the equilibrium entails borrowing along an upward sloping portion of the supply of credit, the limited and costly availability of credit would change the patterns of the deficit throughout the business cycle. The next section will identify the characteristics of such an equilibrium.



(I)
Supply of credit with random future output

(II)
Efficient interest rate and future productivity
(for $B_1 = 0.1$)

Figure 1 Drawn for a uniform distribution, $\delta_2 = 0.3, R = 1.2, Y_0 = 1, G = 0.2, D_1 = 0, r^* = \rho = 0.1, \lambda = 1.25, \alpha = 0.6$

4. TAXES, BORROWING AND THE BUSINESS CYCLE

We evaluate now the way endogenous borrowing constraints impact on borrowing and on the tax rate throughout the business cycle. To facilitate discussion we simulate our model for a time separable CRRA utility:

$$\frac{\{C_1\}^{1-R}}{1-R} + \frac{\{C_2\}^{1-R}}{(1+\rho)(1-R)}; \qquad R \geq 0 \tag{19}$$

To gain further insight, we evaluate now how varying degrees of tax efficiency impact on the borrowing and taxes throughout the cycle. This is done with the help of several simulations, reported in Figures 2–6. Appendix B provides the details of these simulations.

4.1 Limited integration of capital markets – costly tax collection

Our benchmark is an economy where borrowing constraints are not binding, as is the case if the tax capacity of the economy is large relative to the outstanding fiscal liabilities (for the given creditors' bargaining power). This case is illustrated by Figure 2, Panel I, where the bold curve KL depicts the

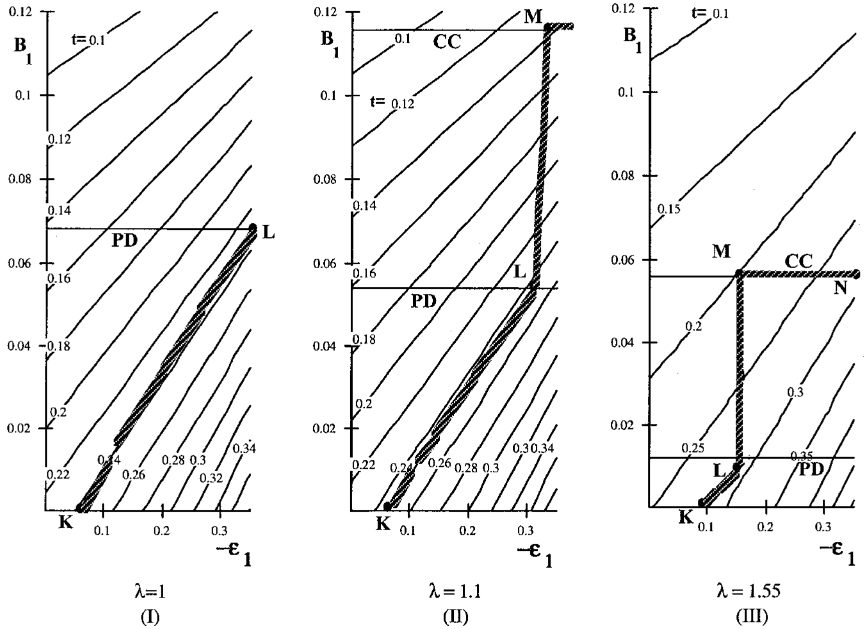


Figure 2 Adverse shock-debt curve for varying tax efficiency (λ). Drawn for $\delta_2 = 0.3$, $R = 1.2$, $Y_0 = 1$, $G = 0.2$, $D_1 = 0$, $r^* = \rho = -0.1$, $\alpha = 0.5$, for the case of a uniform distribution, and a quadratic tax function (see Appendix B for further details). The thick line reports the optimal borrowing. The contours correspond to the first-period tax rate.

dependency of borrowing on the first period adverse shock ($-\varepsilon_1$) for $\lambda = 1$ and $0 \leq -\varepsilon_1 \leq 0.35$. The contours correspond to the first period tax rate. Starting with zero initial debt, lower first period output increases the burden of financing the given fiscal outlays. Hence, it is beneficial to spread the burden intertemporally via borrowing. The tax rate in Figure 2, Panel I exhibits weak cyclical patterns, whereas borrowing exhibits strong counter-cyclical patterns. For example, around point K a 1 percent drop of the GDP increases borrowing by 0.25 percent of the GDP, and induces a small drop of the tax rate by 0.04 percent. Hence, the bulk of the adjustment is accomplished via borrowing. Curve PD depicts the debt level associated with partial default in some states of nature. In panel I, country risk factors are absent, as the optimal debt is below PD.

Panel II considers the case where country risk becomes relevant because the tax collection is relatively inefficient ($\lambda = 1.1$). In comparison to panel I, around point K the tax rate is independent of the present GDP, whereas borrowing is counter cyclical (although less than in panel I). Curve CC depicts the credit ceiling facing the country. At point L we observe a regime switch from a no default to a partial default regime. The switch is associated with a 'borrowing boom' (in the sense that the elasticity of borrowing with respect to the shock goes up) which leads to a relatively large drop in the tax rate in period 1. Once we reach the credit ceiling (point M, panel II, Figure 2), a further drop in the first period income leads to a large increase in taxes.

The rationale for the borrowing boom associated with partial default follows from the drop in the cost of borrowing induced by the switch from a no default to a partial default regime. In the absence of default, marginal borrowing increases repayment in *all* future states of nature by an equal amount. The convexity of the tax collection costs and the diminishing marginal utility imply that raising a net revenue of one dollar is more expensive in bad states of nature, and this cost increases with the revenue needs. Hence, increasing borrowing increases the cost of servicing the debt in period 2 in a non-uniform manner, which is loaded towards the bad states of nature. This, in turn, increases the cost of borrowing, mitigating thereby the marginal borrowing induced by a further increase in the adverse shock. The switch to a partial default regime makes the debt a state contingent liability. It caps the repayment in bad states, implying that the marginal cost of a new debt accrues only in good states. This, in turn, reduces the cost of borrowing, as both the marginal utility of consumption and the cost of raising a one dollar net tax revenue are lower in good states of nature (in comparison to bad states of nature).

Further insight can be gained by studying the factors explaining the change in borrowing elasticity with respect to the adverse productivity shock at the regime switch (at point L in panel II, Figure 2). Let us denote by $x\,|_{pd,L}$ and $x\,|_{n,L}$ the values of a variable x at point L in the partial default and the no default regimes, respectively, and by $y\,|_{\varepsilon=-\delta}$ the value of function y at $\varepsilon=-\delta$. It can be shown that at point L (panel II, Figure 2)

$$\frac{\dfrac{d\log B_1}{d\log(-\varepsilon_1)}\Big|_{pd,L} - \dfrac{d\log B_1}{d\log(-\varepsilon_1)}\Big|_{n,L}}{\dfrac{d\log B_1}{d\log(-\varepsilon_1)}\Big|_{pd,L}} = \frac{\dfrac{(1+r^*)^2}{1+\rho}\dfrac{2\lambda}{\alpha}\left[u'(c_2)(1+\Gamma'(\xi_2))|_{\varepsilon=-\delta_2} - \displaystyle\int_{-\delta_2}^{\delta_2}\{u'(c_2)(1+\Gamma'(\xi_2))\,f\}(\varepsilon)d\varepsilon\right]}{-V''_{B_1B_1}\,|_{n,L}} \tag{20}$$

where $V''_{B_1B_1}$ is the second derivative of the expected utility V with respect to the debt.[10] The term $\int_{-\delta_2}^{\delta_2}\{u'(c_2)(1+\Gamma'(\xi_2))\,f\}(\varepsilon)d\varepsilon$ is the *expected* utility cost of raising one dollar net tax revenue in the second period, whereas $u'(c_2)(1 + \Gamma'(\xi_2))|_{\varepsilon=-\delta_2}$ is the utility cost of raising one dollar net tax revenue at the *worst* state of nature in the second period. Alternatively, $\int_{-\delta_2}^{\delta_2} \{u'(c_2)(1 + \Gamma'(\xi_2))\} f(\varepsilon) d\varepsilon$ is the *average cost* of raising taxes in the second period, whereas $u'(c_2)(1 + \Gamma'(\xi_2))|_{\varepsilon=-\delta_2}$ is the *marginal cost* of raising taxes at the *worst* state of nature in period 2. The diminishing marginal utility and the convex tax collection costs imply that the average cost is lower than the marginal cost at the worst state, and hence

$$u'(c_2)(1 + \Gamma'(\xi_2))|_{\varepsilon=-\delta_2} - \int_{-\delta_2}^{\delta_2} \{u'(c_2)(1 + \Gamma'(\xi_2))\} f(\varepsilon) d\varepsilon > 0 \qquad (21)$$

Following the regime switch (at point L, panel II in Figure 2) we observe partial default at the worst state of nature, $\varepsilon = -\delta_2$. The switch from the no default to the partial default regime alleviates the need to finance the marginal debt in the worst states of nature, where the cost of raising funds is higher $[u'(c_2)(1 + \Gamma'(\xi_2))|_{\varepsilon=-\delta_2}]$. Instead, the marginal debt is financed only in the states where no partial default occurs (at an expected cost of $\int_{-\delta_2}^{\delta_2} \{u'(c_2)(1 + \Gamma'(\xi_2))\} f(\varepsilon) d\varepsilon$). Equation (21), the difference between these two terms measures the drop in the borrowing cost induced by the regime switch. The increase in the borrowing elasticity induced by the regime switch (20) is proportional to the drop in the borrowing cost (21). Note that the gain from the regime switch increases with the concavity of preferences and the convexity of the tax collection costs, as both increase (21). Furthermore, (20) implies that, *ceteris paribus*, greater tax inefficiency (higher λ) increases the borrowing boom (as measured by the increase in the borrowing elasticity).

To confirm this observation we plot in Figure 2, panel III the case where country risk factors are more prevalent as the tax collection is very expensive ($\lambda = 1.55$). A more inefficient tax collection leads to the deterioration of the credit worthiness of the country, which in turn induces the regime switch at a lower debt/adverse shock configuration, increasing the borrowing boom associated with the switch (as measured by the elasticity of the debt with respect to the adverse shock). The deterioration of the tax system shifts both CC and PD downwards, while shifting upwards the contours corresponding to a given tax rate. The deterioration in credit worthiness reduces the total level of debt available to the country. Furthermore, it leads to a discontinuity in the capital market – at point L (panel III, Figure 2) we observe an abrupt regime switch, inducing a large capital inflow that pushes borrowing to the credit ceiling (point M, panel III, Figure 2), leading to a drop in the tax rate. Once that point M is reached, a further drop in the present output does not modify borrowing (as the credit ceiling has been reached). The fiscal needs are met by relatively large tax increases, moving towards point N. While a 1% drop in the GDP at point K (panel III) increases the tax rate by about 0.05 percent, a similar drop of the GDP at point M increases the tax rate by about 0.4 percent!

4.2 Overview – country risk and taxes

We can apply the above results to evaluate the impact of various degrees of tax collection inefficiency on the pattern of borrowing and taxes. Inspection of the three panels in Figure 2 reveals that, in the absence of country risk, the tax rate may be either weakly pro- or counter-cyclical. A greater inefficiency of the tax system will induce greater counter cyclical patterns, as it reduces the flexibility of adjustment.[11] When the credit ceiling facing the country binds, we observe strong counter cyclical patterns of the tax rate (i.e. tax rates increase significantly in recessions). In the transition from a full integration of capital markets to a binding credit ceiling, the tax rates exhibit strong pro-cyclical patterns. The range of pro-cyclical patterns declines as country risk factors are more prevalent due to a less efficient tax collection. Hence, the tax rate exhibits strong counter cyclical patterns in economies operating close to the credit ceiling, whereas the tax rate exhibits very little cyclical patterns in economies operating on the elastic portion of the supply of credit (and hence country risk factors are absent). Economies that frequently alternate between a high debt, binding credit ceiling and a low debt, full credit market integration, are exhibiting unstable patterns – regime switches may be associated with spells of strong pro-cyclical patterns of the tax rate.

Inefficient tax systems (high λ) are characterized by financial fragility – adverse shocks would lead to a region where small changes in fundamentals cause large changes in debt. For example, in Panel III, Figure 2 we observe debt discontinuity at the regime switch, leading us from point L to M. The jump of indebtedness does not impact welfare in our economy, as the expected utility at points L and M are identical (in fact, the equality of welfare defines the value of ε_1 associated with the regime switch). While, in our framework, the increase of indebtedness is inconsequential, it may be of obvious concern when there are externalities (presently not captured in our model), where swings in the debt may further destabilize the economy. A necessary condition for financial fragility is a low tax capacity compared to the government expenditure, and limited bargaining power of creditors.

Figure 3 studies the implications of the borrowing boom on the consumption patterns at the regime switch identified in panel III. This is done by plotting the first and second period consumption as a function of the future productivity. Figure 3, Panel I corresponds to the low debt, no default regime (points L in Panel III, Figure 2). Figure 3, Panel II corresponds to the high debt, default regime (points M in Panel III, Figure 2). The dashed curve corresponds to the first period consumption. Note that the regime switch tilts the intertemporal pattern of consumption against the future. Its consequences on intertemporal consumption smoothing are mixed: the gap between the first

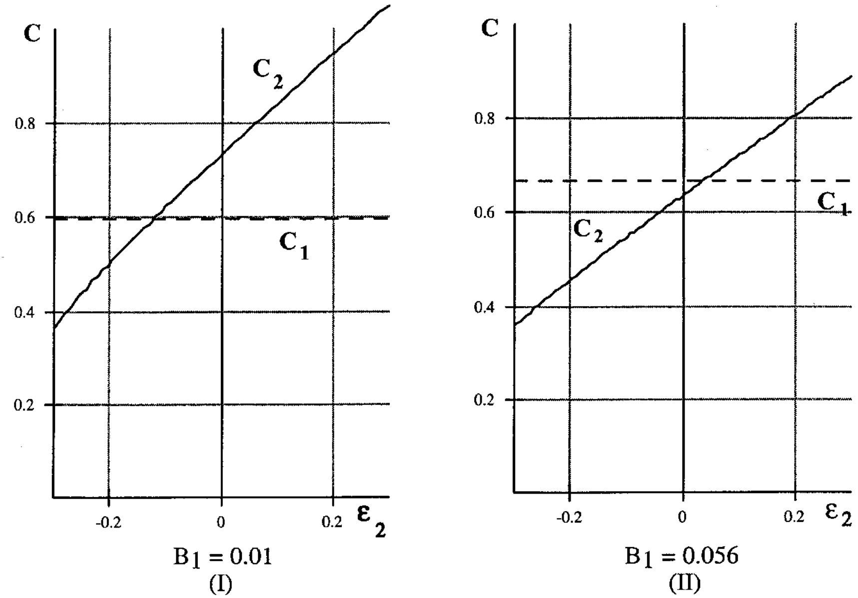


Figure 3 Borrowing boom and intertemporal consumption: comparison of consumption patterns at points L and M, Figure 2, Panel III. Drawn for $\delta_2 = 0.3$, $\varepsilon_1 = -0.159$, $R = 1.2$, $Y_0 = 1$, $G = 0.2$, $D_1 = 0$, $r^* = \rho = 0.1$, $\lambda = 1.55$, for the case of a uniform distribution, and a quadratic tax function (see Appendix B for further details)

period and the second period consumption goes down if the second period output will be high, and goes up if the second period output will be low.

The above analysis focused on the role of the limited tax capacity in determining the shape of the debt curve. One should keep in mind, however, that the integration with the capital market is determined by the interaction among four factors – the tax capacity, the outstanding debt, the fiscal expenditure and the creditors' bargaining power. Figure 2 focused on the implications of varying the tax capacity, holding all other given factors. Similar analysis applies if one is altering one of the other factors. Figure 4 illustrates this point for the case where one varies creditors' bargaining power.[12] An interesting result of our analysis is that the switch from a no default to a partial default regime reduces the cost of borrowing as it alleviates the cost of servicing the debt in bad states of nature. This in turn encourages borrowing, and reduces taxes in the short run.

4.3 Low integration of financial markets and credit ceilings

We turn now to study the degree to which policies that increase creditors' bargaining power (like increasing openness, greater FDI, etc) are beneficial. The above discussion suggests that there are two forces at work. When the country is at the credit ceiling (point M and beyond), greater creditors'

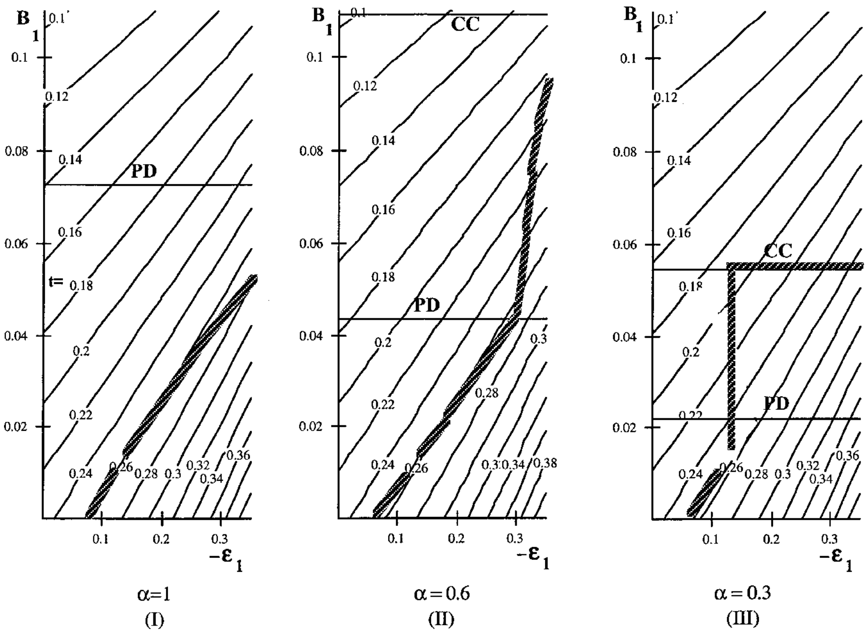


Figure 4 Adverse shock-debt curve for varying creditors bargaining power (α). Drawn for $\delta_2 = 0.3$, $R = 1.2$, $Y_0 = 1$, $G = 0.2$, $D_1 = 0$, $r^* = \rho = 0.1$, $\lambda = 1.25$, for the case of a uniform distribution, and a quadratic tax function (see Appendix B for further details). The thick line reports the optimal borrowing. The contours correspond to the first-period tax rate.

bargaining power would alleviate the borrowing constraint, inducing benefits – the magnitude of which depends on the severity of the shortage of credit. On the other hand, greater creditors bargaining power would lead to higher repayment in states of nature associated with partial default, mitigating some of the benefits of partial default.[13] If the credit ceiling does not bind, the beneficial effect is absent, hence one presumes that increasing creditors' bargaining power is welfare reducing to the developing country. Indeed, applying our analysis one gets that:

$$\frac{\mathrm{d}\,E\{U\}}{\mathrm{d}\,\alpha} = \frac{\partial\, E\{U\}}{\partial B_1}\frac{\partial B_1}{\partial \alpha} + \frac{\partial\, E\{U\}}{\partial \alpha} = \tag{22a}$$

$$-\int_{-\delta_2}^{\varepsilon}\left[\frac{Y_0+\varepsilon}{2\lambda} - G\right]\left[\frac{1}{1+\rho}\,u'(c_2)(1+\Gamma'(\xi_2))\right] f(\varepsilon)\mathrm{d}\varepsilon < 0 \qquad \text{if } \delta_2 > \varepsilon^* > -\delta_2$$

$$\frac{1}{1+r^*}\int_{-\delta_2}^{\delta_2}\left[\frac{Y_0+\varepsilon}{2\lambda} - G\right]\left[u'(c_1)(1+\Gamma'(\xi_1)) - \frac{1+r^*}{1+\rho}\,u'(c_2)(1+\Gamma'(\xi_2))\right] f(\varepsilon)\mathrm{d}\varepsilon \qquad \text{if } \delta_2 = \varepsilon^* \tag{22b}$$

Equation (22a) corresponds to the case of an internal equilibrium, where the credit ceiling is not binding (but the risk premium is positive). The envelope theorem implies that the welfare effect of changing borrowing is zero (as $\frac{\partial E\{U\}}{\partial B_1} = 0$). In these circumstances greater creditors' bargaining power (α) leads to a larger repayment in bad states of nature, in turn leading to $\frac{d\, E\{U\}}{d\, \alpha} < 0$.[14]

Equation (22b) corresponds to the case where the credit ceiling is binding. In these circumstances the first order condition for optimal borrowing is not operative, as we observe a corner solution. The welfare effect of greater creditors power equals the expected increase in repayment multiplied by the difference between the cost of public funds in the first and the second period. A more acute shortage of funds in period one increases this difference, implying that for severe cases of credit rationing $\frac{d\, E\{U\}}{d\, \alpha} > 0$.[15]

5. THE DEBT/VOLATILITY CURVE

We evaluate now how volatility affects borrowing in a recession. One may view this as the other side of precautionary savings – optimal borrowing in the presence of volatility. Figure 5 reports the volatility–debt curve for the case where the output in period 1 is constant, below the expected future output. Panel I reports the debt curve for the case where $\lambda = 1.25$ (KLM). Curve PD depicts debt/volatility configurations associated with a switch from a no default to a partial default regime. The horizontal contours report the tax rate in period 1. Starting with low debt, higher volatility increases the cost of servicing the debt in bad states of nature, thereby reducing borrowing. The regime switch frees a constraint – higher borrowing does not impact upon the repayment in bad future states, only in good ones. This, in turn, reduces the cost of borrowing, inducing an elastic increase in borrowing as the economy switches to the partial default regime. By-products of the higher debt are a drop in first period taxes, a rise in first period consumption, and a consequent drop in second period consumption. For high enough volatility, the front loading of consumption is superior to the attempt to fully service the debt.[16] Panel II (Figure 5) reports the debt curve for the case where the credit ceiling is not binding, as would be the case if the tax system is more efficient (smaller λ). Note that, in the absence of partial default, the debt curve is concave and downward sloping.

Formally, it can be shown that at the regime switch (point L, panel I)

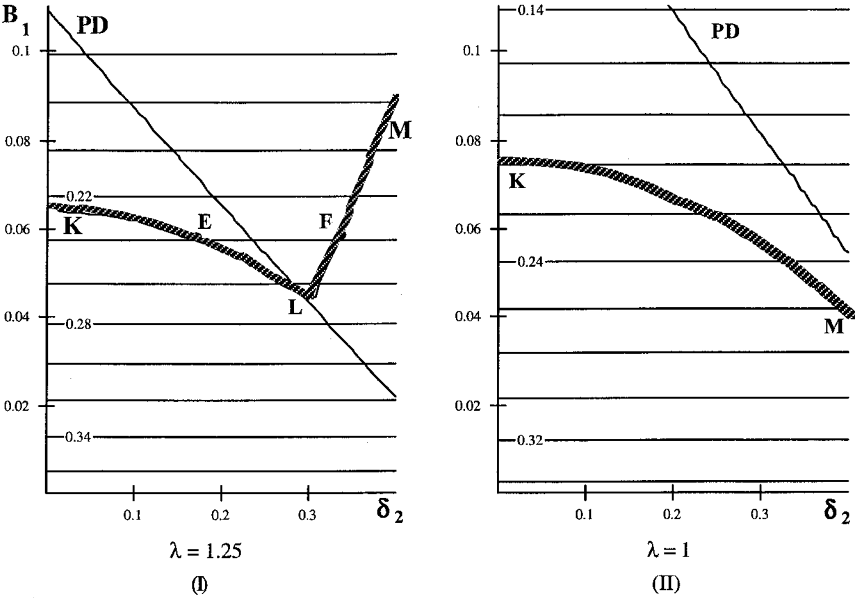


Figure 5 Volatility–debt curve, constant first period output. Drawn for $\varepsilon_1 = -0.3$, $R = 1.2$, $Y_0 = 1$, $G = 0.2$, $D_1 = 0$, $r^* = \rho = 0.1$, $\alpha = 0.6$, for the case of a uniform distribution, and a quadratic tax function (see Appendix B for further details). The thick line reports the optimal borrowing. The contours correspond to the first-period tax rate.

$$\left.\frac{dB_1}{d\delta_2}\right|_{pd,L} = \frac{\frac{1+r^*}{(1+\rho)2\delta_2}\left[\int_{-\delta_2}^{\delta_2}\frac{u'(c_2)(1+\Gamma'(\xi_2))}{2\delta_2}\,d\varepsilon - u'(c_2)(1+\Gamma'(\xi_2))|_{\varepsilon_2=\delta_2}\right]}{-V''_{B_1B_1}|_{pd,L}} > 0 \tag{23a}$$

$$\left.\frac{dB_1}{d\delta_2}\right|_{n,L} = \frac{\frac{1+r^*}{(1+\rho)\delta_2}\left[\int_{-\delta_2}^{\delta_2}\frac{u'(c_2)(1+\Gamma'(\xi_2))}{2\delta_2}\,d\varepsilon - 0.5[u'(c_2)(1+\Gamma'(\xi_2))|_{\varepsilon_2=\delta_2} + u'(c_2)(1+\Gamma'(\xi_2))|_{\varepsilon_2=-\delta_2}]\right]}{-V''_{B_1B_1}|_{n,L}} \tag{23b}$$

The increase in borrowing induced by higher volatility in the partial default regime is proportional to the difference between the *average cost* of raising taxes in the second period and the *marginal cost* of raising taxes in the *best* state of nature in the second period, as reported by (23a). The rationale for this result is that greater volatility shifts the effective repayment for a given debt from the states where no default occurs (the entire support at point L) to the best state of nature. This burden reallocation induces a drop in the cost of serving the debt as given by the difference between the average and the marginal cost evaluated at the best state of nature. The concavity of preferences and the convexity of the tax collection cost imply that this difference is positive, increasing with the curvature of preferences and the tax collection cost.

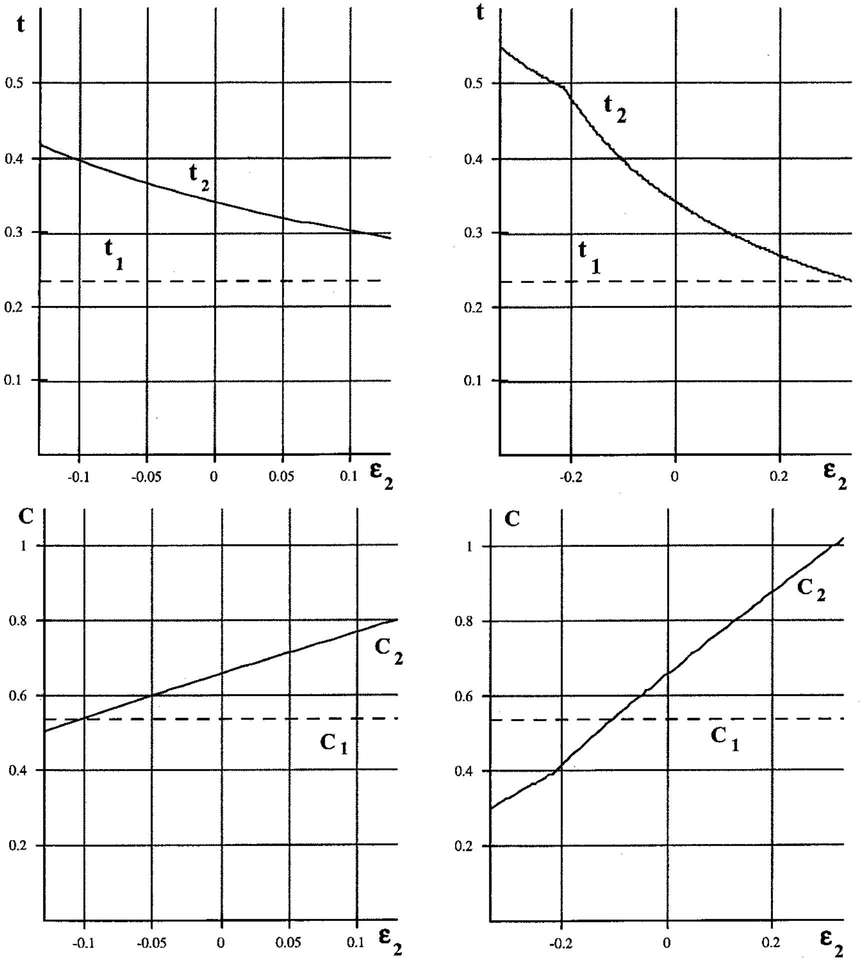
In the absence of default, a higher volatility spreads the repayment over a larger support, leading to a change in the cost of debt given by the *average cost* minus the *average* of the *marginal cost* in the *best* and the *worst* states of nature, as reported by (23b). If the marginal cost of raising taxes is strongly convex (as is in our case) the net effect is that higher volatility reduces borrowing in the absence of partial defaults.

The regime switch induced by sovereign risk implies that the same debt level may be observed with both low and high volatility, as is the case in points E and F in Figure 5, panel I. Figure 6 contrasts the pattern of first and second period taxes (top panels) and consumption (lower panels) between the low and the high volatility regime (as depicted by points E and F in Figure 5, panel I). While the first period consumption is the same in both regimes, the high volatility regime induces a much higher volatility of second period consumption, and a more pronounced counter cyclical pattern of taxes. The partial default has the effect of reducing the second period tax rate in the worst states of nature, thereby mitigating the drop in consumption observed in these states.

6. CONCLUDING REMARKS

Barro's (1979) tax smoothing results apply as long as capital markets are well functioning, as is the case when the tax capacity is relatively large. Our paper shows that, for a developing country whose tax capacity is relatively small and its GDP is volatile, the optimal tax rate is state contingent. In the presence of sovereign risk, partial defaults would occur in bad states of nature, leading to a risk premium and a credit ceiling. Partial defaults change the nature of the debt contract to a state contingent one, inducing a 'burden shifting' from bad to good states of nature. This burden shifting reduces the cost of borrowing, implying that a switch from a no default to a partial default regime is associated with a borrowing boom. With partial default, optimal borrowing smoothes intertemporally the expected cost of raising taxes *only* between the states of nature associated with *full* repayment. The switch to a partial default regime is associated with financial fragility, where small changes in fundamentals (like a drop in the foreign interest rate, an increase in government expenditure, etc) may lead to a large accumulation of debt.

Country risk modifies considerably the pattern of taxes and borrowing in recessions. In the absence of country risk, the tax rate exhibits weak cyclical patterns, where a drop in output leads to higher borrowing, smoothing intertemporally the cost of raising taxes. With limited access to the international credit market, a large enough adverse output shock leads to a switch



$\delta_2 = 0.133$ $\delta_2 = 0.337$

Figure 6 Taxes, consumption, and second period productivity. Drawn for $\varepsilon_1 = -0.3$, $R = 1.2$, $Y_0 = 1$, $G = 0.2$, $B_1 = 0.06$, $r^* = \rho = 0.1$, $\lambda = 1.25$, $\alpha = 0.6$, for the case of a uniform distribution, and a quadratic tax function (see Appendix B for further details)

to the partial default regime. The switch induces a large capital inflow that pushes borrowing to the credit ceiling. The regime switch front-loads consumption and back-loads taxes. A further drop in output leads to large increases in taxes – with a binding credit ceiling, we observe a strong counter-cyclical pattern of the tax rate. In the transition from full integration of capital markets to a binding credit ceiling, the tax rates exhibit strong pro-cyclical patterns. The range where pro-cyclical patterns are observed declines as country risk factors are more prevalent. Hence, the tax rate exhibits strong counter-cyclical patterns in economies operating close to the credit ceiling, whereas the tax rate exhibits very little cyclical patterns in economies operating on the elastic portion of the supply of credit (where country risk factors are absent). Economies that alternate frequently between the high debt, binding credit ceiling regime and the low debt and no risk premium regime exhibit unstable patterns – regime switches are associated with spells of strong pro-cyclical patterns of tax rates and large counter-cyclical borrowing.

Borrowing constraints are shown to increase the volatility of taxes, as well as the volatility of debt. A country whose tax capacity is limited, its income is volatile, and its government expenditure is relatively large, would 'choose' the high debt, partial default regime as a logical consequence of its structure. The likelihood that a country will operate in the high debt regime increases the less efficient its tax collection is, the larger is the fiscal expenditure, and the lower is its integration with the international capital market (e.g. due to lower openness). Debt relief in the high debt regime, without structural changes that enhance the tax capacity or reduce fiscal expenditure, may be of limited consequences. It may induce a regime switch to a low debt regime, but then small shocks would push the country back to the high debt regime.

Our model defines a metric for evaluating the relative size of debt and country risk. It is shown that holding the debt/GDP ratio constant, a country is more risky the higher is its volatility, the greater its fiscal commitments relative to the efficiency of its tax system, and the lower its integration with the international credit market, as measured by the enforceability of debt contracts. Our analysis shows that the welfare effect of policies that increase creditors' bargaining power (like greater openness) is favourable for countries facing the credit ceiling and encountering severe scarcity of funds, but is detrimental for countries that operate on the upward sloping portion of the supply of credit although below the credit ceiling.

In order to facilitate the analysis, we chose to ignore several important considerations. We close the paper with an overview of possible extensions. Our analysis simplified matters by assuming only two periods. Its key results, however, are applicable for the general case, of *n* periods (up to the modifications discussed below). Key factors inducing state contingent taxes are the borrowing constraints, as they prevent the tax smoothing allowed by borrowing. The assumption of only two periods truncates the horizon in the second period, as there is no way to engage in tax smoothing in the final period. This has the consequence of exacerbating the responsiveness of the tax rate in period 2 to output shocks in that period, as it imposes an extra credit ceiling – no new borrowing in period 2. Extending the horizon beyond 2 periods will eliminate this restriction. Thus, in an infinite horizon model, taxes will be less responsive to shocks *as long as* the credit ceiling is not binding. This extension will not modify the state contingent nature of taxes in those states of nature where the credit ceiling is binding, or in those states of nature where risk premium exists. Hence, the key feature of our framework is the introduction of endogenous credit constraints, which truncates the ability to shift the tax burden intertemporally, requiring state contingent taxes.

An additional simplifying assumption is that the shocks impacting the tax base follow a uniform distribution. Some of the results, like the possibility of discontinuity of debt in the regime switch, are clearly driven by the specific distribution. Yet, the key result of the paper – events that lead to a higher probability of partial default will increase the elasticity of borrowing, hold to other distributions as well. This result is driven by the fact that a higher probability of partial debt repayment shifts the burden of repayment from bad to good future states of nature, reducing thereby the utility cost of borrowing, leading to a borrowing boom.

We focused on the representative agent's welfare, thereby ignoring issues related to the distribution of income and to political economy considerations. This assumption, however, may be of limited validity if the patterns of taxation and access to the global capital market differ for capital and labour. In these circumstances, one should go well beyond the focus on efficiency in order to understand the implications of the fiscal deficits and the associated international borrowing.

Our paper followed Below and Rogoff (1989), assuming that the threat of sanctions supports an equilibrium with partial default. Allowing for reputation considerations may provide an alternative way to explain partial default (although Obstfeld and Rogoff (1996, Section 6.1) suggest that this mechanism may not be re-negotiation-proof). One may expect the main results of our model to apply in alternative models explaining partial defaults, as long as the effective repayment increases with the realized output in the range of partial defaults.

The international capital market was modelled as risk free. Our model can be extended to the more realistic case, where both the domestic and the international markets are risk averse. In such a model, the arbitrage condition linking the domestic and the international market will include 'risk premium' terms reflecting the full covariance structure between the domestic and the foreign market. Hence, the financial spread between the international 'risk free' and the emerging market interest rates will compensate for both the default risk due to country risk factors, and the risk associated with exposure to an asset that is not fully correlated with the global market. The present model should be viewed as a special case of such an extended framework, where risk aversion of the global investor is negotiable.

Moral hazard, in the form of implicit or explicit loan guaranties, has been viewed as an important explanatory factor accounting for external debt build-up. Moral hazard tends to subsidize external debt, in ways that are determined by the source of the insurance. If the credit guarantees are provided by the lender's tax payer (what is referred to as international moral hazard), then it will induce a more elastic supply of funds facing the emerging market, potentially relieving the contour cyclical tendency of taxes, at a cost to the lender's tax payer. Alternatively, if the loan guarantees are financed by the emerging market, it will tend to magnify the budgetary crunch induced by adverse shocks, intensifying the counter cyclical patterns.

APPENDIX A

This Appendix generalizes (7) for the case where the initial outstanding debt is positive, and derives (17).

The supply of credit and outstanding debt

Suppose that the outstanding debt in period 1 is positive, given by D_1. We assume that the maximum resources that can be 'confiscated' by creditors in period i is $\alpha[T_i^* - G]$. Consequently, the effective ceilings on net resource transfers to creditors are

$$S_1 - B_1 \leq \alpha\,[T_1^* - G] \qquad (6')$$

$$S_2 = min[(1 + r)\,B_1;\;\; \alpha\,[T_2^* - G]$$

where S_i stands for the (gross) debt repayment to foreign creditors in period i. Applying (6′), the expected net present value of the maximum repayment is $\alpha\{T_1^* - G\} + \frac{E[\alpha\{T_2^* - G\}]}{1 + r*}$. If the outstanding initial debt exceeds this expression, no new voluntary debt will be extended in period 1 (as the expected repayment of the new debt is zero). The country defaults partially in period 1, transferring $\alpha[T_1^* - G]$ in that period. The remaining debt is 'rescheduled' for period 2, when it will be repaid up to the realized ceiling:

$$\alpha\{T_1^* - G\} + \frac{E\,[\alpha\,\{T_2^* - G\}]}{1 + r*} \leq D_1 \Rightarrow \begin{bmatrix} S_1 = \alpha[T_1^* - G] \\ B_1 = 0 \\ S_2 = min\{\alpha[T_2{}^* - G];\,(1 + r*)(D_1 - \alpha[T_1{}^* - G])\} \end{bmatrix} \qquad (7'a)$$

If the outstanding initial debt falls short of the expected net present value of the maximum resource transfer, new voluntary debt will be extended in period 1.

The equilibrium with voluntary borrowing is summarized by

$$\alpha\{T_1^* - G\} + \frac{E\,[\alpha\,\{T_2^* - G\}]}{1 + r^*} > D_1 \Rightarrow \begin{bmatrix} 1. & S_1 = D_1 \\ 2. & B_1 < \dfrac{E[\alpha\{T_2^* - G\}]}{1 + r^*} \\ 3. & S_2 = min\{\alpha[T_2^* - G]; B_1(1 + r)]\} \\ 4. & B_1(1 + r^*) = E\{S_2\} \end{bmatrix} \quad (7'\text{b})$$

where E is the expectation operator, r is the contractual interest rate in period 1, and $(S_1; S_2)$ is the first and second period debt repayment. The net repayment in the first period is $D_1 - B_1$, where B_1 is the voluntary new debt, granted in period 1 at a contractual interest rate r. The new debt cannot exceed the expected discounted value of the second period repayment ceiling (condition (7′b.2)). The contractual debt will be repaid in the second period up to the repayment ceiling (condition (7′b.3)). The interest rate r is determined by the condition that the expected yield on the new debt equals the risk free interest rate r^*.[17]

Equation (17)

We describe the derivation of (17). To simplify presentation, let us denote by $\tilde{u}(\xi)$ the utility when the tax needs GDP ratio is ξ, $\tilde{u}(\xi) = u(Y[1 - \xi - \Gamma(t(\xi))])$. Optimal borrowing is obtained by

$$\underset{B_1}{Max} \left[u(\xi) + \frac{1}{1+\rho} \int_{-\delta}^{\varepsilon^*} \tilde{u} \left(\frac{\alpha\left[\dfrac{Y_0 + \varepsilon}{2\lambda} - G\right] + G}{Y_0 + \varepsilon} \right) f(\varepsilon) \mathrm{d}\,\varepsilon + \int_{\varepsilon^*}^{+\delta} \tilde{u} \left(\frac{B_1(1 + r) + G}{Y_0 + \varepsilon} \right) f(\varepsilon) \mathrm{d}\,\varepsilon \right] \quad (\text{A1})$$

Leading to the first order condition

$$u'(c_1)\{1 + \Gamma'(\xi_1)\} = \frac{1}{1+\rho} \int_{\varepsilon^*}^{\delta} u'(c_2)\,\{1 + \Gamma'(\xi_2)\} f(\varepsilon)\mathrm{d}\varepsilon \, \frac{\mathrm{d}\{B_1(1 + r)\}}{\mathrm{d}B_1} \quad (\text{A2})$$

Recalling (9′), $\dfrac{\mathrm{d}\{B_1(1 + r)\}}{\mathrm{d}B_1} = \dfrac{1 + r^*}{\int_{\varepsilon^*}^{\delta} f(\varepsilon)\mathrm{d}\varepsilon}$. Hence,

$$u'(c_1)\{1+\Gamma'(\xi_1)\} = \frac{1+r^*}{1+\rho}\int_{\varepsilon^*}^{\delta} u'(c_2)\,\{1+\Gamma'(\xi_2)\} f(\varepsilon)\mathrm{d}\varepsilon \,\frac{1}{\int_{\varepsilon^*}^{\delta} f(\varepsilon)\mathrm{d}\varepsilon} \tag{A3}$$

Alternatively,

$$u'(c_1)\{1+\Gamma'(\xi_1)\} = \frac{1+r^*}{1+\rho} E\{u'(c_2)\{1+\Gamma'(\xi_2)\} \mid \varepsilon > \varepsilon^*\} \tag{A4}$$

APPENDIX B

In this Appendix we review the simulations reported in Figures 2–6. The thick shaded curve in Figures 2, 4 and 5 traces the optimal borrowing as a function of various parameters (the first period shock in Figures 2 and 4, and the volatility in Figure 5). These simulations are performed for the case where shocks are following a uniform distribution; the tax function is quadratic, (3′); the consumer's utility is (19); and $Y_1^0 = Y_2^0 = Y_0$; $\lambda \geq 1$. The assumption of a uniform distribution allows us to trace the closed form solution of the supply of funds, reported by (10). The assumption of a quadratic tax function provides us with a closed form solution for the consumption as a function of borrowing. We summarize this solution below. The optimal borrowing is obtained by solving (16). Specifically, applying equations (3′) and (13) we infer that, for $\xi_i \leq \frac{1}{2\lambda}$ and $1 \leq \lambda$,

$$\Gamma(t(\xi_i)) = \frac{1}{2\lambda}\left[1-\sqrt{1-2\lambda\xi_i}\,\right]^2 \tag{B1}$$

Applying (B1), (11) and (15) we infer that

$$C_1 = Y_1\left(1-\frac{G-B_1}{Y_1}-\frac{1}{2\lambda}\left[1-\sqrt{1-2\lambda\,\frac{G-B_1}{Y_1}}\,\right]^2\right) \tag{B2}$$

$$C_2 = \begin{cases} (Y_0+\varepsilon)\left(1-\dfrac{B_1(1+r)+G}{Y_0+\varepsilon}-\dfrac{1}{2\lambda}\left[1-\sqrt{1-2\lambda\,\dfrac{B_1(1+r)+G}{Y_0+\varepsilon}}\,\right]^2\right) & \text{if}\quad \varepsilon^* < \varepsilon \\ (Y_0+\varepsilon)\left(1-\dfrac{\alpha\left[\dfrac{Y_0+\varepsilon}{2\lambda}-G\right]+G}{Y_0+\varepsilon}-\dfrac{1}{2\lambda}\left[1-\sqrt{1-2\lambda\,\dfrac{\alpha\{(Y_0+\varepsilon)/2\lambda-G\}+G\}}{Y_0+\varepsilon}}\,\right]^2\right) & \text{if}\quad \varepsilon^* > \varepsilon \end{cases} \tag{B3}$$

where the interest rate r in (B3) is given by (10). Applying (B2), (B3) and (10) to (19) we infer the value of the expected utility maximized in (16) as a function of the first period borrowing. The optimal borrowing is obtained by solving the corresponding first-order condition, (17). While there is no close form solution to this first-order condition, the simulations in Figures 2–6 summarize the characteristics of the optimal solution.

NOTES

This paper is part of the NBER's research program in International Trade and Investment. Any opinions expressed are those of the authors and not those of the NBER, Warburg, or the IDB. We would like to thank the very useful comments of two anonymous referees. Any errors are ours.

1 On the United States see Barro (1979, 1986). On European economics see Barro (1987).

2 This is not due to differences in the persistence of typical shocks to the tax base; Gavin and Perotti (1996) show that the persistence of output growth in Latin America is very similar to those observed in the industrial countries, and that the persistence of shocks to the terms of trade (an important determinant of the tax base in some countries) is actually lower in Latin America than in the industrial economies.

3 In Mexico, the budget moved toward surplus during 1995, despite a 7 percent decline in real GDP. In Argentina, the March program contemplated a 2 percentage point increase in the fiscal surplus, but a deeper than expected recession meant that the budget moved from rough balance in 1994 to a deficit of somewhat less than one percent of GDP in 1995. For further discussion and references on the Mexican crisis see Sachs *et al.* (1995) and Calvo and Mendoza (1996).

4 The debt contract with default is a very restrictive form of the public debt contracts with state-contingent returns that were analysed in Chari *et al.* (1994) and discussed in Barro (1995). Our reason for focusing on this restrictive form of state-contingency is empirical relevance: with the quantitatively minor exception of some commodity-indexed bonds, returns on government debt are never state contingent except through the possibility of default when conditions deteriorate. (We set aside here the issue of inflation and domestic currency debt, which raises additional complications. See Calvo and Guidotti, 1993 and Barro, 1995.)

5 For further discussion and references on sovereign risk see Krugman (1985), Edwards (1985), Dooley (1988), Helpman (1989), Bulow and Rogoff (1989), Frenkel *et al.* (1989), and Calvo and Kaminsky (1991) and Edwards (1993). See also Eaton and Fernandez (1995) for an overview of the literature on external debt.

6 Note that the tax capacity (5) also depends on the state of nature, which determines the actual output. To simplify notation we suppress occasionally the time or the state index.

7 Hence, α captures all the factors that may influence the integration of capital markets beyond the tax capacity. An example illustrating our model is a country where the tax revenue relies heavily on export of a commodity, and G measures fiscal spending needed to support production. In these circumstances the repayment may be reduced to the one summarized in (6). Further analysis regarding the factors impacting α can be found in Bulow and Rogoff (1989), who derive endogenously the debt sustainable in a Rubinstein re-negotiation game. Applying their reasoning, greater openness (defined by greater dependence on trade), more effective sanctions by creditors (seizure of goods or assets, as well as elimination of trade credit), and less patient

debtors and more penitent creditors will tend to increase α. The Appendix to Bulow and Rogoff (1989) also provides an overview on the efficacy of legal sanctions, arguing that the threat of trade sanction can plausibly explain the actual repayments that do occur. Our framework follows this logic, assuming that cross default clauses in loans from banks and provisions for the organization of bondholders' committees provide the mechanism coordinating the actions of lenders when default occurs.

8 Applying (3) and the definition of ξ we infer that $\xi Y_i = T_i = Y_i[t_i - \Gamma]$. Thus, $t_i = \xi_i + \Gamma_i$, and $C_i = Y_i(1 - t_i) = Y_i(1 - \xi_i - \Gamma_i)$.

9 Note that raising one dollar of net taxes increases the gross tax bill by $1 + \Gamma'(\xi_1)$. Borrowing one dollar increases the first period utility by the product of the gross tax saving times the marginal utility.

10 Note that the second-order condition for maximization implies that $V''_{B_1B_1} < 0$.

11 Applying (12) it follows that $\frac{d t_1}{d Y_1} = -\text{sign}\left[\xi_1 + \frac{dB_1}{dY_1}\right]$, where $\frac{dB_1}{dY_1} < 0$. Hence, a drop in the first period output would increase the first period tax if $\xi_1 > -\frac{dB_1}{dY_1}$. Higher inefficiency of the tax system reduces the responsiveness of borrowing, leading to a counter-cyclical tax pattern for a high enough λ. Indeed, with full integration of capital market, for $\lambda = 1$ we observe a weak pro-cyclical pattern of the tax rate, for $\lambda = 1.1$ the tax rate is practically constant throughout the cycle, and it is counter-cyclical for $\lambda = 1.55$.

12 Note that because we focus in Figure 4 on a relatively costly tax collection system the tax rate observed with full integration of capital markets is weakly counter-cyclical.

13 Recall that the benefit from partial default stems from capping repayment in bad states of nature. Greater creditors' bargaining power alleviates this cap.

14 Note that $\frac{\partial E\{U\}}{\partial \alpha} = -\int_{-\delta_2}^{\varepsilon^*}\left[\frac{Y_0 + \varepsilon}{2\lambda} - G\right]\left[\frac{1}{1+\rho}u'(c_2)(1 + \Gamma'(\xi_2))\right]f(\varepsilon)d\varepsilon$.

15 If the credit ceiling is binding

$$\frac{\partial E\{U\}}{\partial B_1} = u'(c_1)(1 + \Gamma'(\xi_1)); \quad \frac{\partial B_1}{\partial \alpha} = \frac{\int_{-\delta_2}^{\delta_2}\left[\frac{Y_0 + \varepsilon}{2\lambda} - G\right]f(\varepsilon)d\varepsilon}{1 + r^*},$$

and $\frac{\partial E\{U\}}{\partial \alpha} = -\int_{-\delta_2}^{\delta_2}\left[\frac{Y_0 + \varepsilon}{2\lambda} - G\right]\left[\frac{1}{1+\rho}u'(c_2)(1 + \Gamma'(\xi_2))\right]f(\varepsilon)d\varepsilon$. Equation (22b) is obtained by collecting the various terms in the above expressions.

16 No default commitment requires a low first-period debt because of the inability to repay in future bad states of nature.

17 Note that by borrowing $B_1 > D_1 - \alpha[T_1^* - G]$ the repayment constraints (6) are satisfied. This borrowing plan is feasible because $D_1 - \alpha\{T_1^* - G\} \le \frac{E[\alpha\{T_2^* - G\}]}{1 + r^*}$.

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