Asset Supply and Private Information in Over-The-Counter Markets*

Zachary Bethune
University of Virginia

Bruno Sultanum
Federal Reserve Bank of Richmond

Nicholas Trachter
Federal Reserve Bank of Richmond

October 2016
Preliminary - Do not distribute

Abstract

This paper studies asset supply and trading dynamics in a decentralized market with incomplete information. New assets are issued in a primary market through efficient auctions and then trade amongst investors who have heterogeneous valuations in an over-the-counter (OTC) secondary market. The innovation regarding the OTC market literature is that, in all trades, investors' valuations for the asset are private information. The innovation regarding the literature on trade under private information is that the distribution of investors' valuations is endogeneous—it depends on trading dynamics in the OTC secondary market. We calibrate the model to match features of the U.S. municipal bond market between 2005-2014 and examine the effects of private information on key financial market indicators such as trade volume, asset issuance, secondary market liquidity, and welfare.

KEYWORDS: Decentralized markets, mechanism design, efficient auctions.

*Contact: Zachary Bethune: zab2t@virginia.edu, Bruno Sultanum: bruno@sultanum.com, Nicholas Trachter: nicholas.trachter@rich.frb.org. This paper previously circulated under the title "Decentralized Trade with Private Values". We thank Jim Albrecht, Julien Hugonnier, Ben Lester, Susan Vroman, Pierre-Olivier Weill, Randall Wright, and seminar participants at Georgetown, UC Santa Barbara, the Summer Workshop on Money, Banking, and Asset Markets at FRB-Chicago, SED Toulouse, Wisconsin School of Business Money, Banking, and Asset Markets Conference, and the Cornell/Penn State Macroeconomics Conference for helpful suggestions and feedback. Any opinions or views expressed in this article are not necessarily those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

1 Introduction

[CHANGE]

The purpose of this paper is to study the effects of private values on trade in decentralized, over-the-counter (OTC) asset markets. OTC markets are ubiquitous for both real and financial assets (e.g. real estate, federal funds, municipal bonds, credit derivatives, etc.) A unifying feature of these markets is that trade occurs in bilateral or multi-lateral meetings. One of the strongest results in economics, the Myerson and Satterthwaite (1983) impossibility theorem, states that if parties lack common knowledge of the gains from trade, there is no arrangement that guarantees ex-post efficient outcomes. Under private information, it is always in the interest of the buyer to pretend they have a low valuation for the asset being sold in order to move the price down in negotiation. In a similar way, the seller's interest is to pretend they have a high valuation for the object in order to move the price up. Hence, both parties require informational rents. When the gains from trade are not sufficient to cover these rents, trade breaks down. In this paper, we ask how this inefficiency caused by private values at the bilateral level impacts aggregate trading outcomes and market efficiency.

Our environment contains many relevant features of OTC markets. Assets are created and issued through a primary market in which one issuer faces many potential investors with unknown and heterogeneous valuations. Issuers form ex-ante efficient auctions, at a cost, by randomly sampling investors. Hence, issuers take into consideration the distribution of investor valuations when creating and selling assets. After initial issuance, assets trade in a secondary OTC market in which investors meet bilaterally over time subject to search frictions in the style of Duffie et al. (2005) or Lagos and Rocheteau (2009). In a meeting, asset owners make take-it-or-leave-it offers to non-owners taking into consideration that their valuation is uncertain and private information. Investors are heterogeneous in their flow utility of holding assets, given by an arbitrary underlying distribution as in Hugonnier et al. (2014). This heterogeneity create gains from trade and generally represents the fact that private valuations for assets, even with a known and common dividends, can differ both across investors (and potentially through time).[1]

[1]For instance, heterogeneous private valuations can represent differing liquidity and hedging needs by dealers or customers trading financial assets, or moving and income shocks by consumers trading houses.

As a result of these utility/liquidity differences, investors value assets not only for their discounted stream of dividends but also for their liquidity in the secondary market.

Private values distort trade along three dimensions. First, for any given distribution of investor asset valuations, the probability of trade within a meeting is distorted. Under complete information, agents engage in trade whenever the non-owner values the asset more than the owner. However, trade under private values requires that the valuation of the non-owner be larger than the owner's valuation *plus* a term that captures the informational rents possessed by the non-owner. This is the impossibility result of Myerson (1981). When the gains from trade in a meeting are not larger than the informational rent, no trade occurs. Second, private values affect the endogenous equilibrium distribution of investor valuations. Since trades between investors with close valuations are destroyed, in equilibrium there are relatively more investors with low valuation that are holding assets and relatively more investors with high valuations that are not holding assets. In other words, asset mis-allocation increases in the aggregate. This in turn further distorts trade at the bilateral level as it is easier for any buyer (even with high valuation) to pretend they have a low valuation. Finally, private values decreases the issuance of assets as the liquidity value to investors in the secondary market falls.

We quantify these effects by calibrating the model to match features of the municipal bond market in the United States, described in Green et al. (2007). The municipal bond market in the US is large, with an average of $400 billion a year in new issuance, and features an active OTC secondary market that averages 40,000 trades per day at an average par value of $14 million.[2] Further, municipal bonds feature low default risk and are not likely to be affected by common value problems, such as adverse selection.[3] In our preliminary calibrations, we find that private values have a significant affect on trading outcomes in the municipal bond market. Compared to the full information benchmark, private information lowers the supply of municipal bonds by 22%, decreases aggregate trade volume by 78%, and decreases total welfare by 8%. Further, we find that restoring

[2]Data on aggregate trading activity in the municipal bond market is available through the Municipal Securities Research Board's (MSRB) Electronic Municipal Market Access (EMMA) website, http://emma.msrb.org.

[3]The average one-year default rate on Moody's rated municipal bonds from 1970 to 2014 was 0.015%. Source: "US Municipal Bond Defaults and Recoveries, 1970-2014", Moody's Investor Service Credit Policy Memo, July 25, 2015.

information in primary markets has little effects on the allocation or welfare, however restoring information in the secondary municipal bond market nearly achieves the complete information benchmark

Our paper is related to both the micro and macro literature on trade with private information. At a micro-level, our trading mechanism follows early work by Myerson (1981) and Myerson and Satterthwaite (1983) that models trade of an indivisible asset under private information about agents' valuations. These papers, and the literature that followed, are largely interested bilateral efficiency while taking the distribution of valuations as exogenous. Alternatively, we provide a framework where the distribution of valuations is endogenous and responds to frictions, both at a bilateral level (e.g. information frictions) and at an aggregate level (e.g. search frictions, supply/demand effects). The macro-finance literature on trade under private information has largely focused on uncertainty about common values, such as asymmetric information and adverse selection problems. Adverse selection has been used to understand asset market liquidity in both centralized markets (e.g. Eisfeldt, 2004; Chari et al., 2014; and Bigio, 2015) and decentralized markets (e.g. Guerrieri et al., 2010; Chang, 2014; and Chiu and Koeppl, 2015). Alternatively, we consider private information about private values, a relevant component of the valuation of assets in which the quality of the asset is easily verified, and analyze the effects on both secondary market liquidity and primary market issuance. Cujean and Praz (2015) study an environment with asymmetric information about private values in which asset holdings are unrestricted. Finally, Zhang (2016) also considers asymmetric information about privates values but in an environment in which traders can form long-term relationships. We abstract from long-term relationships.

The paper is structured as follows. Section 2 illustrates the aggregate effects of private information in a simple, static model of trade under private information. Section 3 then introduces the general environment, defines the steady state equilibrium and proves its existence. Section 5 describes the calibration and contains our main experiments. Finally, Section 6 concludes.

2 Simple model

We start our analysis with a simple, static example of bilateral trade between issuers of assets and investors. Trade occurs in a decentralized market, where investors 'valuation of the asset—what we refer as their type—are private information. We use this simple model to illustrate the effects of private information on market outcomes such as the aggregate level of assets, ask prices and welfare.

2.1 Environment

There is one period, a measure one of agents called issuers and a measure one of agents called investors. There is a non-divisible asset and utility is transferable. Assets are created by issuers and creating an asset entails incurring a cost, c. Issuers do not value holding assets.[4] Investors can hold either zero or one unit of the asset and receive a utility flow ν from holding it. Investors' utility flow from holding the asset is private information, and identically and independently distributed according to a distribution F. The distribution F has a continuous density f, support $[\underline{\nu}, \bar{\nu}]$ and $f(\nu) > 0$ for all ν in the support. To keep things as simple as possible, we let $c = \underline{\nu}$. Given no heterogeneity on c, in the simple model we only have private information on the investor side.

Issuers and investors meet bilaterally and, for simplicity, we assume that no issuer or investor stays unmatched. When an issuer meets an investor, they trade using a mechanism designed to maximize the issuer's expected welfare. This structure gives all the Pareto weight in the meeting to the issuer (in the appendix we develop a more general version that allows for any Pareto weights). The advantage of using the current formulation is that it is analytically more tractable and also is directly decentralized using take-it-or-leave-it (TIOLI) offers.

By the revelation principle, it is without loss of generality to consider only direct mechanisms. A direct mechanism is composed of two functions, $p : [\underline{\nu}, \bar{\nu}] \to [0,1]$ and $x : [\underline{\nu}, \bar{\nu}] \to \mathbb{R}$, that maps the investor's type into the probability of trade and the transfer of utility. After the trade occurs, agents receive utility flows and the period ends.

[4] One could alternatively, interpret c as the issuer's opportunity cost of giving up the asset.

2.2 The optimal direct mechanism

The optimal direct mechanism is the solution to problem

$$\max_{p,x} \int [x(v) - p(v)c] \, dF(v) \tag{1}$$

subject to

$$IR: \quad vp(v) - x(v) \geq 0 \tag{2}$$

$$IC: \quad vp(v) - x(v) \geq vp(\hat{v}) - x(\hat{v}) \; . \tag{3}$$

for all v, $\hat{v}$. Problem (1) maximizes the expected profit of an issuer subject to individual rationality and incentive compatibility (equations 2 and 3). Myerson (1981) considers a more general mechanism design problem of a single seller faced with N buyers. We focus on the case with N equal to one for simplicity and, in the appendix, develop a more general version that allows the issuer to run auctions with multiple buyers.

Individual rationality (constraint 2) says that the issuer cannot force the investor to trade. Incentive compatibility (constraint 3) says that the issuer cannot prevent an investor from misrepresenting their type. Constraints (2) and (3) guarantee that truth-telling is a Bayesian-Nash equilibrium associated with the game induced by the direct mechanism that solves (1). In what follows we assume investors are playing the truth-telling equilibrium. Let $h(v) = f(v)/[1 - F(v)]$ be the hazard function of F. As in Myerson (1981), we call the problem *regular* if $v - 1/h(v)$ is increasing. For the remainder of this section, we assume that the problem is *regular*.[5]

Using integration by parts and the incentive compatibility constraint, this problem can be reduced to finding the probability of trade that maximizes

$$\max_{p} \int p(v) \, [v - 1/h(v) - c] \, dF(v) \tag{4}$$

subject to $p(v)$ being non-decreasing in v.[6] The unconstrained maximum of problem (4)

[5] A sufficient condition is that $h'(v) > 0$, a feature of many common probability distributions such as the Normal, Pareto, and Exponential distributions. If this condition is not satisfied, a solution still exists, but is slightly more complicated.

[6] See Myerson (1981) for details.

is attained by

$$p(v) = \begin{cases} 1 & \text{if } v - 1/h(v) \geq c \\ 0 & \text{otherwise} \end{cases}. \tag{5}$$

Since the problem is regular, the above $p(v)$ is non-decreasing in v and, as a result, it also attains the constrained maximum.

The probability of trade is determined by the investor's true utility from holding the asset, v, minus an informational rent, $1/h(v)$. The term $v - 1/h(v)$ is labeled in the literature as the virtual valuation of the investor. It measures how much the issuer values selling the asset to type v investors.[7] The informational rent, $1/h(v)$, depends on the underlying distribution of investors valuations F. A high hazard $h(v)$ at v implies a high virtual valuation of type v investor, which occurs because either the density of investors at v, $f(v)$, is high, or because the measure of investors with valuation above v, $1 - F(v)$, is low. A high value of $f(v)$ leads to a high virtual valuation because it implies that there is a large amount of investors at v, so that being able to sell to these investors is highly desirable for the issuer. However, selling to investors of type v implies giving informational rents to investors of type above v, that has measure $1 - F(v)$.

The solution to this problem implies a cut-off rule in which an issuer only sells the asset to investors of type at or above v^*. The cut-off v^* is implicitly defined by

$$v^* - 1/h(v^*) = c. \tag{6}$$

The assumptions on F guarantee that the cut-off v^* exists and $v^* \in (\underline{v}, \bar{v})$. Further, the mechanism transfers satisfy

$$x(v) = \begin{cases} v^* & \text{if } v \geq v^* \\ 0 & \text{otherwise,} \end{cases} \tag{7}$$

which says that (i) transfers only occur upon trade and (ii) any non-owner above the cutoff receives the asset but pays the valuation of the investor at the cutoff. The optimal

[7] With complete information there is no need for the issuer to give up informational rent, so we can think of the virtual valuation in a complete information model as being simply v.

mechanism can be decentralized with issuers making take-it-or-leave-it offers (TIOLI). In this case, the problem of the issuer is to choose an ask price a that maximizes

$$\max_a \, (a - c) \left[1 - F(a)\right] \; .$$

The first order condition reads $1 - F(a) = f(a)(a - c)$, and provides an implicit expression for the ask price, $a = c + 1/h(a)$. Under the regularity condition, *i.e.* $\nu - 1/h(\nu)$ being increasing in ν, this equation has a unique solution and we get that $a = \nu^*$. Therefore, the issuer issues an asset when $\nu \geq a = \nu^*$, which is the same issuance probability of equation (5), and charges $a = \nu^*$, which is the same transfer of equation (7).

Figure 1: Trade and no-trade regions

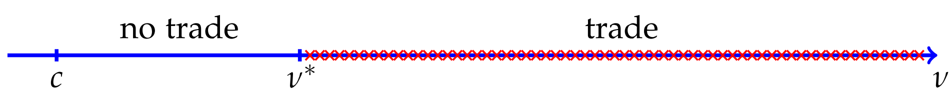


Figure 1 displays the trade region associated with the optimal direct mechanism. Under incomplete information, trade is not ex-post efficient because $\nu^* > \underline{\nu} = c$. That is, trades where $\nu \in (c, \nu^*)$ are lost and only meetings involving an investor with valuation above ν^* are ex-post efficient. This is in contrast with a complete information environment, where trade is ex-post efficient. Under complete information, the optimal mechanism has the issuer charging investors exactly their valuation. As a result, all meetings result in trade since investors valuation $\nu \in [\underline{\nu}, \bar{\nu}]$ are always above the issuance cost $c = \underline{\nu}$.

2.3 Example using the Pareto distribution

Below, we characterize the effects of private information on trade volume, the expected profit of issuers, and welfare under the assumption that ν is distributed as Pareto. That is, $F(\nu) = 1 - (\underline{\nu}/\nu)^\alpha$, where $\underline{\nu} > 0$ is the scale parameter of the distribution (and also the lower bound of the support) and $\alpha > 2$ is the shape parameter. The assumption that $\alpha > 2$ guarantees that the results from this section hold even though the Pareto distribution is not defined in a compact support. The Pareto distribution has many nice features that allow for a fully analytical solution. In particular, it is the case that the hazard function can be solved in closed form: $h(\nu) = \alpha/\nu$. Then, the investor's virtual valuation becomes $\frac{\alpha-1}{\alpha}\nu$, which is linear in the investor's type, and the cut-off value is $\nu^* = \frac{\alpha}{\alpha-1}c$. We study how informational rents depend on α and show how these rents affect trade volume, the creation of assets, and the gains from trade.

Using the TIOLI offers decentralization for the mechanism, we have that an issuer sets an ask price $a = \frac{\alpha}{\alpha-1}c$. When selling the asset, the issuer incurs the cost of production c, and sets the markup at $\frac{\alpha}{\alpha-1}$. Note that, when using the Pareto distribution, α is the price elasticity of demand since $\frac{-d[1-F(a)]}{da} \cdot \frac{a}{1-F(a)}$ equals α.

2.3.1 The aggregate level of assets: Trade volume and asset supply

Aggregate trade volume is a key indicator of the functioning of financial markets. Moreover, in this simple model, trade volume accounts for the total level of assets in the economy. This occurs because every trade requires an issuer to issue a new asset. In this section we show how the level of assets depends on the extent of private information, captured by the parameter α. Note that α also captures the degree of dispersion of investors' valuations—high α is associated with low variance of valuations. Trade volume is defined as

$$TV = \int p(\nu) dF(\nu) \ . \tag{8}$$

The integral term is the fraction of meetings that results in trade, where the probability of trade in a meeting is given by $p(\nu)$, as defined in equation (5). Using the solution for the mechanism when F is Pareto, trade volume is given by

$$TV = \int \mathbb{1}_{\{\nu_n \geq \frac{\alpha}{\alpha-1}c\}} dF(\nu_n) = 1 - F\left(\frac{\alpha}{\alpha-1}c\right) = \left(\frac{\alpha-1}{\alpha}\right)^{\alpha} \in \left(\frac{1}{4}, \frac{1}{e}\right) \ .$$

We can similarly define trade volume under complete information, TV^{CI}. Under complete information, as we discussed before, every meeting ends up in trade because investors valuations are always above the issuance cost. Therefore,

$$TV^{CI} = \int dF(\nu_n) = 1 \ .$$

Because trade volume equals one under complete information, TV also measures the distortion in trade volume—and thus on the level of asset supply—due to incomplete information. The trade distortion is always positive, bounded between $1/4$ and $1/e$ ($\approx$ 0.37), and increasing in α (decreasing in variance). The fact that the distortion does not vanish as the variance goes to zero is noteworthy. When α goes to infinity, the probability that the type of the investor is not close to the type of the issuer becomes negligible. However, the remaining investors with high valuation relative to the issuer have an informational rent high enough to distort trade, even in the limit as α goes to infinity.

2.3.2 Gains from trade

The distortion affecting the trade volume also affects the gains from trade accrued by the issuer, which we denote by GT_i. Using that F is Pareto, we have that

$$GT_i = [1 - F(\nu^*)](\nu^* - c) = \left(\frac{\alpha - 1}{\alpha}\right)^{\alpha} \frac{c}{\alpha - 1} .$$

We can similarly define the gains from trade accrued by the issuer under complete information, which is $GT_i^{CI} = \int (\nu - c) dF(\nu) = \frac{c}{\alpha - 1}$. Thus, the distortion that incomplete information generates on gains from trade for issuers is

$$\frac{GT_i}{GT_i^{CI}} = \left(\frac{\alpha - 1}{\alpha}\right)^{\alpha} = \frac{TV}{TV^{CI}} .$$

As with trade volume, the distortion reduces as the extent of the informational friction becomes smaller. And, as it happened with trade volume, the gains from trade of issuers are always distorted, even when the friction vanishes (α becomes arbitrarily large). The gains from trade for investors, which we denote by GT_n, is given by

$$GT_n = \int_{\nu^*} (\nu - \nu^*) dF(\nu) = \frac{c}{\alpha - 1} \left(\frac{\alpha - 1}{\alpha}\right)^{\alpha - 1} .$$

Under complete information, the gains from trade accrued by investors is zero. This occurs because issuers can charge investors exactly their valuation for the asset if they know what this valuation is, while they have to pay an informational rent to guarantee truth-telling when they don't know it.

The share of the total gains from trade accrued by issuers is given by

$$\frac{GT_i}{GT_i + GT_n} = \frac{\alpha - 1}{2\alpha - 1} \in \left[\frac{1}{3}, \frac{1}{2}\right] .$$

The share of gains from trade that are captured by issuers increases as the informational friction becomes smaller (that is, when α increases). However, the share is bounded away from one due to the same logic behind the bound for trade volume.

Finally, notice that total gains from trade become zero as the informational friction becomes negligible. That is, $\lim_{\alpha\to\infty} GT_i + GT_n = 0$. Under complete information, the total gains from trade, which are simply the gains from trade of issuers, also converges to 0. That is, $\lim_{\alpha\to\infty} GT_i^{CI} = 0$. This result states that, as the informational friction vanishes, the limiting behavior of the gains from trade in the incomplete information economy is the same than the limiting behavior of the first-best, complete information economy. Although we concluded before that for the incomplete information economy, trade volume and the gains from trade accrued by issuers are always downward distorted from their complete information counterpart, the measure of these losses is arbitrarily small, in the sense that trades that do not occur because of the friction were likely to involve agents with close valuations, and thus with arbitrarily small gains from trade. As a result, the distortions in trade volume and gains from trade by issuers do not translate into distortions in the overall gains from trade in the limit.

To summarize, the informational friction leads to a distortion in trade volume, total asset supply, and the gains from trade. The extent of these distortions depend on the pervasiveness of private information, captured by the parameter α in our example with the Pareto distribution. Trade volume and gains from trade by issuers are always distorted, even when there is little dispersion in types. Total gains from trade under incomplete information are also distorted. However, the distortion vanishes as dispersion in types becomes small. All these effects will be present in our larger, dynamic model. In the dynamic model, informational rents will affect welfare in two main ways. First, by distorting trade, it generates asset misallocation. Second, by distorting the gains from trade of issuers, it reduces the incentives to issue new assets, thus generating a sub-optimal level of assets in the steady state, with important effects on trade and welfare.

3 General model

In this section, we introduce the full model of decentralized trade. We are guided by many relevant features in OTC markets. Assets are issued in a primary market where one seller faces many potential buyers. These assets are valued not only at their discounted stream of cash flows, but also for their liquidity in secondary markets. Secondary market trade occurs bilaterally and is subject to search frictions. Finally, investors differ in their valuation of an asset and these valuations are private information.[8]

3.1 Environment

Time is continuous and infinite. There is a unit measure of infinitely-lived and risk-neutral investors and a measure one of infinitely-lived issuers who all discount the future at rate $r > 0$. Investors' asset holdings are discrete, either zero or one. As before, we label investors holding the asset as *owners*, and those not holding the asset as *non-owners*. There is transferable utility across all agents, investors and issuers. An investor receives utility flow ν from holding an asset. We refer to ν as the investor's type. The investor's type ν is private information. Types ν are distributed according to F, with support $[\underline{\nu}, \bar{\nu}]$ and density f, where $f(\nu) > 0$ for all ν in the support.[9] It is straightforward to extend the setup to allow for time varying types, in the style of Duffie et al. (2005). In their setup, as in many in the literature, preference shocks are important in order to guarantee that trade occurs in the steady state. In our setup, given that assets mature -and new assets are created- we do not need this assumption.

New assets are created by issuers, and outstanding assets are destroyed every period with Poisson arrival rate μ. Issuers do not value holding assets and they have random

[8]There are many reasons for investors' valuations of an asset to differ, such as different liquidity needs, regulatory constraints or hedging needs due to risk management.

[9]This is a model of private values. Private values for an asset are distinguished from common values, in which the types of some agents convey information of the valuation of other agents. See Fundenberg and Tirole (1991), Chapter 7 for a discussion of how the information structure affects the mechanism design problem and Krishna (2010) for a detailed analysis of auctions under differing information structures.

opportunities to create new assets. In particular, they receive an i.i.d. investment opportunity (i.e. the option to create an asset) at Poisson rate $\lambda_a > 0$. Creating an asset entails incurring in a cost c, which we assume to be random across issuers and time, drawn from a distribution $G(c)$ with density $g(c)$ and support $[\underline{c}, \bar{c}]$, where $g(c) > 0$ for all costs c in the support. Upon the realization of an investment opportunity, an issuer contacts an investor randomly.[10] If the issuer decides to create the asset, they incur cost c. Note that the issuer always has the option not to issue the asset, in which case, no cost is incurred. For instance, this would happen if the issuer does not reach an agreement to sell the asset to the investor he is in contact with.

Investors meet each other with Poisson arrival rate $\lambda_b/2$ and they can trade the asset created by the issuer. The primary market consists of trades between issuers and investors, and the secondary market consists of trades between investors.

We restrict our attention to steady state analysis so we do not include time t in the set of states of the economy. Let s denote the fraction of investors holding an asset, let $\Phi_o(\nu)$ denote the measure of owners with type ν or below, and $\Phi_n(\nu)$ denote the measure of non-owners of type ν or below. Let $V_i(\nu)$ be the value function of an investor of type $i \in \{o, n\}$, where o stands for *owner* and n stands for *non-owner*, with current valuation ν, and let $\Delta(\nu) = V_o(\nu) - V_n(\nu)$. We refer to $\Delta(\nu)$ as the reservation value of an investor. It is the maximum price a non-owner investor of type ν is willing to buy the asset for, or the minimum price an owner of type ν is willing to sell his asset for. Finally, let V_c denote the value function of an issuer before the issuance opportunity and before knowing his cost of producing the asset.

3.2 Primary market trade mechanism

The primary market consists of trades between issuers and non-owners. As in the simple model, we consider trading mechanisms that maximize the seller's - in this case the issuer's - expected gains from trade.[11]

Without loss of generality we focus on direct mechanisms, given by a pair of functions $x^a : [\underline{c}, \bar{c}] \times [\underline{\nu}, \bar{\nu}] \to \mathbb{R}$ and $p^a : [\underline{c}, \bar{c}] \times [\underline{\nu}, \bar{\nu}] \to \mathbb{R}$. As before, $x^a(c, \nu_n)$ represents the

[10] It is easy to extend this assumption so that the issuer can contact several investors at once, in which case he runs an auction instead of trading bilaterally.

[11] The model can easily be generalized in order to allow for different Pareto weights.

transfer from the non-owner to the issuer when the issuer has issuance cost c and the non-owner has type ν_n. The value $p^a(c, \nu_n)$ represents the probability of transferring the asset when the issuer has issuance cost c and the non-owner has type ν_n. Since the mechanism maximizes the issuer's gains from trade, it must solve

$$\max_{p^a, x^a} \int [x^a(c, \nu_n) - p^a(c, \nu_n)c] \, d\frac{\Phi_n(\nu_n)}{1-s} \tag{9}$$

subject to

$$IR: \quad \Delta(\nu_n)p^a(c, \nu_n) - x^a(c, \nu_n) \geq 0 \tag{10}$$

$$IC: \quad \Delta(\nu_n)p^a(c, \nu_n) - x^a(c, \nu_n) \geq \Delta(\nu_n)p^a(c, \hat{\nu}_n) - x^a(c, \hat{\nu}_n) \tag{11}$$

for all c, ν_n and $\hat{\nu}_n$. Given a mechanism x^a and p^a, we have that the expected gains from trade of an issuer and a non-owner in a meeting in the primary market are given by

$$\pi_c^a(c) = \int [x^a(c, \nu_n) - p^a(c, \nu_n)c] \, d\frac{\Phi_n(\nu_n)}{1-s} \tag{12}$$

$$\pi_n^a(\nu_n) = \int [p^a(c, \nu_n)\Delta(\nu_n) - x^a(c, \nu_n)] \, dG(c). \tag{13}$$

The first equation is the expected gains from trade of an issuer with cost c conditional on meeting a non-owner. It takes expectation with respect to ν_n of the transfer from the non-owner, $x^a(c, \nu_n)$, minus the expected cost of issuance, $p^a(c, \nu_n)c$. The second equation is the expected gains from trade of a non-owner with type ν_n conditional on meeting with an issuer. It takes expectation with respect to c of the expected gain of obtaining the asset, $p^a(c, \nu_n)\nu_n$, minus the transfer to the issuer, $x^a(c, \nu_n)$. The gains from trade are evaluated under the assumption that issuers and non-owners truthfully reveal their types to the mechanism. The constraints to the problem ensure that such an equilibrium exists and, in order to keep the presentation simple, we assume that is what agents do instead of explicitly writing the strategic game associated with the mechanism.

3.3 Secondary market trade mechanism

The secondary market consists of trades between owners and non-owners. As in the primary market, we consider mechanisms that maximize the expected gains from trade of owners in the secondary market. A direct mechanism in the secondary market is given by a pair of functions $x^b : [\underline{\nu}, \bar{\nu}]^2 \to \mathbb{R}$ and $p^b : [\underline{\nu}, \bar{\nu}]^2 \to \mathbb{R}$, where $x^b(\nu_o, \nu_n)$ represents the transfer from the non-owner to the owner when the owner has type $\bar{\nu}$ and the non-owner has type ν_n. Similarly, $p^b(\nu_o, \nu_n)$ represents the probability of transferring the asset when the owner has type ν_o and the non-owner has type ν_n. Since the mechanism maximizes the owner's gains from trade, it must solve

$$\max_{p^b, x^b} \int \left[x^b(\nu_o, \nu_n) - p^b(\nu_o, \nu_n)\Delta(\nu_o) \right] d\frac{\Phi_n(\nu_n)}{1-s} + \Delta(\nu_o) \tag{14}$$

subject to

$$IR: \quad \Delta(\nu_n)p^b(\nu_o, \nu_n) - x^b(\nu_o, \nu_n) \geq 0 \tag{15}$$

$$IC: \quad \Delta(\nu_n)p^b(\nu_o, \nu_n) - x^b(\nu_o, \nu_n) \geq \Delta(\nu_n)p^b(\nu_o, \hat{\nu}_n) - x^b(\nu_o, \hat{\nu}_n) \tag{16}$$

for all ν_o, ν_n and $\hat{\nu}_n$. Problem (14) is closely related to problem (1) in the simple model and it can be solved in a similar fashion. However, it has two important differences that make it technically more challenging. First, investors' valuations are not given directly by their type ν_n, but by the reservation value, Δ, which takes into consideration the liquidity value of assets. Second, the distribution of non-owner types, $\Phi_n/(1-s)$, is endogenous and not given by the underlying shock distribution, F.

Given a mechanism x^b and p^b, the expected gains from trade of an owner and a non-owner in a meeting in the secondary market are given by

$$\pi_o^b(\nu_o) = \int \left[x^b(\nu_o, \nu_n) - p^b(\nu_o, \nu_n)\Delta(\nu_o) \right] d\frac{\Phi_n(\nu_n)}{1-s} + \Delta(\nu_o) \;, \tag{17}$$

$$\pi_n^b(\nu_n) = \int \left[p^b(\nu_o, \nu_n)\Delta(\nu_n) - x^b(\nu_o, \nu_n) \right] d\frac{\Phi_o(\nu_n)}{s} \;. \tag{18}$$

The first equation is the expected gains from trade of an owner with type ν_o conditional on meeting with a non-owner. It takes expectation with respect to ν_n of the transfer from the non-owner, $x^b(\nu_o, \nu_n)$, minus the expected opportunity cost of selling the asset, $p^b(\nu_o, \nu_n)\Delta(\nu_o)$, plus his continuation value if the asset is not sold, $\Delta(\nu_o)$. The second equation is the expected gains from trade of a non-owner with type ν_n conditional on meeting with an owner. It takes expectation with respect to ν_o of the expected gain of getting the asset, $p^b(\nu_o, \nu_n)\nu_n$, minus the transfer to the owner, $x^b(\nu_o, \nu_n)$. As before, the gains from trade are evaluated under the assumption that owners and non-owners are truthfully revealing their types to the mechanism.

3.4 The value functions of investors and issuers

The value function of an owner, in its Hamilton-Jacobi-Bellman representation, is

$$rV_o(\nu) = \nu + \mu\big[V_n(\nu) - V_o(\nu)\big] + \lambda_b(1-s)\pi_o^b(\nu) \ . \tag{19}$$

This equation states that the value of owning an asset, discounted at rate r, equals the sum of four terms. The first term accounts for the flow utility of holding the asset, ν. The second term is the change in value when the asset depreciates and the owner becomes a non-owner. The third term accounts for the net gains of an owner meeting a non-owner. An owner meets a non-owner at rate λ_b $(1-s)$, where $1-s$ is the fraction of investors that are non-owners. The value function of a non-owner follows similarly,

$$rV_n(\nu) = \lambda_b s \pi_n^b(\nu) + \lambda_a \pi_n^a(\nu) \ . \tag{20}$$

The value for a non-owner of type ν is the sum of three terms. The first term accounts for the net gains for the non-owner meeting an owner. A non-owner meets an owner at rate $\lambda_b s$, where s is the fraction of investors that are owners. The second term accounts for the net gains accrued by the non-owner when meeting an issuer, an event that occurs at rate λ_a. Using equations (19)-(20) we can compute the reservation value function for investors, $\Delta(\nu)$, according to

$$r\Delta(\nu) = \nu - \mu\Delta(\nu) + \lambda_b(1-s)\pi_o^b(\nu) - \lambda_b s \pi_n^b(\nu) - \lambda_a \pi_n^a(\nu) \ . \tag{21}$$

Finally, the value function for an issuer is given by

$$rV_c = \lambda_a \int (1-s)\pi^a_c(c)dG(c) \ . \tag{22}$$

This equation provides that the flow value of being an issuer equals the arrival rate of an issuance opportunity times the expected profits that the issuer would get if he produces and sells the asset to a non-owner investor.

3.5 The distribution of assets

In this section, we provide equilibrium conditions for the distribution of owners' types, $\Phi_o(\nu)$, the distribution of non-owners' types, $\Phi_n(\nu)$, and for the fraction of agents holding assets in the secondary market, s. The law of motion for $\Phi_o(\nu)$ is given by

$$\dot{\Phi}_o(\nu) = -\mu\Phi_o(\nu) - \lambda_b \int_{\underline{\nu}}^{\nu} \int_{\nu}^{\bar{\nu}} p^b(\nu_o, \nu_n) d\Phi_n(\nu_n) d\Phi_o(\nu_o) + \lambda_a \int_{\underline{c}}^{\bar{c}} \int_{\underline{\nu}}^{\nu} p^a(c, \nu_n) d\Phi_n(\nu_n) dG(c) \ . \tag{23}$$

This equation considers the change in the mass of owners with valuation below ν. The first term on the right-hand-side of the equation accounts for those owners with valuation below ν that become non-owners because the asset matures. The second term accounts for meetings in the secondary market where owners with valuation below ν sell the asset to buyers with valuation above ν. Finally, the third term accounts for meetings where issuers sell the asset to non-owners with valuation below ν. Given the law of motion for $\Phi_o(\nu)$, we can easily obtain an expression for $\Phi_n(\nu)$ using that

$$\Phi_o(\nu) + \Phi_n(\nu) = F(\nu) \ , \tag{24}$$

which simply states that the measure of owners of type below ν plus the measure of non-owners of type below ν has to be equal to the total measure of investors of type below ν. Finally, by definition, all issued assets must be held by owners, or

$$\Phi_o(\bar{\nu}) = s. \tag{25}$$

3.6 Equilibrium

In this section we define an equilibrium and we provide an existence result. We start with the definition of a symmetric, steady state equilibrium.

Definition 1. *A symmetric, steady state equilibrium is a family* $\{\Delta, V_c, \Phi_o, \Phi_n, s, x^a, p^a, x^b, p^b\}$ *such that:*

(i) the value function of an issuer V_c *satisfies equation* (22) *where* π_c^a *is given by equation* (12), *and the reservation value of investors* $\Delta(\nu)$ *satisfies equation* (21) *where* π_n^a, π_o^b *and* π_n^b *are given by equations* (13), (17) *and* (18);

(ii) the measure Φ_o *is such that* $\dot{\Phi}_o = 0$, *where* $\dot{\Phi}_o$ *is defined in equation* (23), Φ_n *satisfies equation* (24) *and the amount of assets in the economy s satisfy* (25); *and*

(iii) the mechanism in the primary market, x^a *and* p^a, *solves problem* (9), *while the mechanism in the secondary market,* x^b *and* p^b, *solves problem* (14).

Notice that the definition of equilibrium does not include the value function V_o and V_n. This occurs because we can solve for the equilibrium solely with the reservation value function Δ and then back out V_o and V_n using (19) and (20).

The existence of a solution to the problem that we analyze here can be complicated. In the static model analyzed in Section 2, existence reduces to finding a solution to the mechanism; this occurs because the distribution of valuations is exogenous. In the dynamic model, for a given distribution of valuations, existence also reduces to finding solutions to the two mechanisms (i.e. primary and secondary markets). However, because the distributions, Φ_o and Φ_n, and reservation value, Δ, are endogenous, existence also involves a fixed point in these objects.

Proposition 1. *There exists a symmetric steady state equilibrium.*

Proof. To be added. □

In Section 2, when we solved the static model we use existing results developed for static setups, particularly Myerson (1981). In order to find a solution for the mechanism design problem, Myerson (1981) assumes a regularity condition over the exogenous distribution of valuations, that is, $\nu - \frac{1}{h(\nu)}$ needs to be increasing in ν. The regularity condition allows for an easy characterization of the optimal mechanism. In the general formulation of the model that we explore here, the required regularity condition is that $\Delta(\nu) - \frac{1-s-\Phi_n(\nu)}{\Delta'(\nu)\phi_n(\nu)}$ is increasing in ν, where $\phi_n(\nu) = \frac{d\Phi_n(\nu)}{d\nu}$.

One problem is that in our general model the regularity condition might not hold. This stems from the fact that Δ and Φ_n are endogenous. When the regularity condition does not hold, Myerson (1981) proposes a solution called ironing. Ironing is a procedure to flatten the regions where $\Delta(\nu) - \frac{1-s-\Phi_n(\nu)}{\Delta'(\nu)\phi_n(\nu)}$ is decreasing in ν, making it constant for ν in those regions. Loosely speaking, this makes the regularity condition hold again. However, this solution can generate jumps for the probability of trade implied by the mechanisms, p^a and p^b. These jumps can translate into discontinuities in the density of the endogenous distributions, $\frac{\Phi_o}{s}$ and $\frac{\Phi_n}{1-s}$, complicating the application of standard fixed point arguments to show existence of an equilibrium.

The idea of the proof of the proposition is the following. We begin by constructing a sequence of distribution functions, valuations and mechanisms, $\{\Phi_o^k, \Phi_n^k, s^k, \Delta^k, x^{ka}, p^{ka}, x^{kb}, p^{kb}\}_k$. The operator we use to construct this sequence updates the measures Φ_o^k and Φ_n^k using polynomials (for example, Chebyshev polynomials) of degree k for the associated densities. This guarantees that the distributions are well behaved through the whole sequence and thus we can apply Myerson's results to generate the $k+1$ term from the k term in the sequence. The next step is to show that the sequence $\{\Phi_o^k, \Phi_n^k\}_k$ is equi-continuous and then, by the Arzelà-Ascoli theorem, it has a subsequence which converges uniformly.[12] The convergence of a subsequence of $\{\Delta^k, x^{ka}, p^{ka}, x^{kb}, p^{kb}\}_k$ follows similar arguments. Hence, we conclude that, passing to a subsequence if necessary, $\{\Phi_o^k, \Phi_n^k, s^k, \Delta^k, x^{ka}, p^{ka}, x^{kb}, p^{kb}\}_k$ converges to a limit $\{\Phi_o^*, \Phi_n^*, s^*, \Delta^*, x^{*a}, p^{*a}, x^{*b}, p^{*b}\}$. The last step in the proof is to show that this limit satisfies the equilibrium conditions.

[12]Note, however, that even though for the entire sequence we have that Φ_o^k and Φ_n^k are continuously differentiable, the limits do not have to be since the space $\mathscr{C}^1$ is not closed.

4 Complete information

An important benchmark for our model is a version where, in any meeting, investors' valuations, ν, are observed. That is, there is complete information. In this case, the seller can extract full surplus from the trade. The trade mechanisms are easy to characterize, given by

$$p^{a,CI}(c,\nu_n) = \begin{cases} 1 & \text{if } \Delta^{CI}(\nu_n) \geq c \\ 0 & \text{otherwise} \end{cases}, \quad x^{a,CI}(c,\nu_n) = \begin{cases} \Delta^{CI}(\nu_n) & \text{if } \Delta^{CI}(\nu_n) \geq c \\ 0 & \text{otherwise} \end{cases}, \tag{26}$$

$$p^{bCI}(\nu_o,\nu_n) = \begin{cases} 1 & \text{if } \nu_n \geq \nu_o \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad x^{bCI}(\nu_o,\nu_n) = \begin{cases} \Delta^{CI}(\nu_n) & \text{if } \nu_n \geq \nu_o \\ 0 & \text{otherwise} \end{cases}. \tag{27}$$

Notice, under complete information trade is always bilaterally ex-post efficient. Issuers create assets if the seller they match with has a larger valuation than their cost and investors trade in the secondary market whenever the non-owner values the asset more than the owner. The equations for the reservation value and distribution of valuations are the same of the incomplete information model but replacing the mechanisms (x^a, p^a, x^b, p^b) with $(x^{aCI}, p^{aCI}, x^{bCI}, p^{bCI})$. That is,

$$\begin{aligned} \dot{\Phi}_o^{CI}(\nu) = & -\mu\Phi_o^{CI}(\nu) \\ & - \lambda_b \int_{\underline{\nu}}^{\nu} \int_{\nu}^{\bar{\nu}} p^{bCI}(\nu_o,\nu_n) d\Phi_n^{CI}(\nu_n) d\Phi_o^{CI}(\nu_o) \qquad (28) \\ & + \lambda_a \int_{\underline{c}}^{\bar{c}} \int_{\underline{\nu}}^{\nu} p^{aCI}(c,\nu_n) d\Phi_n^{CI}(\nu_n) dG(c) \text{ , and} \qquad (29) \end{aligned}$$

$$r\Delta^{CI}(\nu) = \nu - \mu\Delta^{CI}(\nu) + \lambda_b(1-s)\pi_o^{bCI}(\nu) - \lambda_b s \pi_n^{bCI}(\nu) - \lambda_a \pi_n^{aCI}(\nu) \text{ ,} \tag{30}$$

where expected gains from trade are

$$\pi_c^a(c) = \int \left[x^a(c,\nu_n) - p^a(c,\nu_n)c\right] d\frac{\Phi_n(\nu_n)}{1-s} \tag{31}$$

$$\pi_n^a(\nu_n) = \int \left[p^a(c,\nu_n)\Delta(\nu_n) - x^a(c,\nu_n)\right] dG(c). \tag{32}$$

$$\pi_o^b(\nu_o) = \int \left[x^b(\nu_o,\nu_n) - p^b(\nu_o,\nu_n)\Delta(\nu_o)\right] d\frac{\Phi_n(\nu_n)}{1-s} + \Delta(\nu_o) \text{ ,} \tag{33}$$

$$\pi_n^b(\nu_n) = \int \left[p^b(\nu_o, \nu_n)\Delta(\nu_n) - x^b(\nu_o, \nu_n) \right] d\frac{\Phi_o(\nu_n)}{s} \,. \tag{34}$$

The equilibrium definition also follows in the same fashion. We conjecture that a complete information equilibrium always exists, as in the incomplete information case, and it is also unique and implements the first best allocation (that is, the allocation that maximizes aggregate welfare). We have partial results suggesting this is the case, but the proof is still incomplete.

5 Calibration

In this section, we quantitatively study the implications of private information in the US municipal bond market. The US municipal bond market is (i) large, with nearly 40,000 transactions per day at an average par of $14 million, (ii) decentralized – all trades occur bilaterally – and (iii) features low default risk. Low default risk is important in that private information about the payoffs of municipal bonds are not a major friction in this market.

We use data collected and provided by the Municipal Securities Rulemaking Board (MSRB). The data covers all transactions between investors and broker-dealers as well as between broker-dealers from 2005 until 2014. The MSRB requires all broker-dealers to report their municipal securities transactions within 15 minutes of the time of trade. These data are then released with a lag to the public through MSRB's Electronic Municipal Market Access (EMMA) portal.[13]

For each transaction, we observe characteristics about the issue being traded, such as the unique CUSIP identifier, maturity date, coupon rate, and the date when interest began to accrue as well as trade characteristics such as the date and time the trade took place, the price, the par value, and whether or not the trade was a purchase by a dealer from an investor, a sale from a dealer to an investor, or an inter-dealer transaction. In some instances, the yield is reported. Table 1 reports the descriptive statistics. Following Green et al. (2007), we label all trades that take place within the first 90 days from issuance as 'primary market' transactions and those occurring after as 'secondary market'

[13]Historical MSRB data is available to academics through Wharton Research Data Services (WRDS).

transactions.[14]

Table 1: Descriptive Statistics on Municipal Bond Transactions, 2005-2015

	All Transactions	Primary Market	Secondary Market
Observations (millions)	86.40	18.50	67.90
Issues (millions)	1.89	1.26	1.25
Total value (trillion $)	17,701.69	6,840.02	10,861.67
Average par (million $)	0.20	0.37	0.16
Coupon rate (%)	4.52	4.20	4.60
Yield-to-Worst (%)	3.91	3.97	3.90
Maturity (years)	17.83	15.34	18.51
Years until maturity	13.66	15.33	13.20

Note: Sample is all municipal bond transactions reported to the MSRB from 2005-2014. We drop observations on variable rate securities, if missing price, or if the par value traded is less than $1,000. We windsorize the price and yield at 99%. If the yield-to-worst is not reported on the transaction, we use the estimated yield-to-maturity.

Municipal bonds are actively traded after their initial issuance. From 2005-2014, we observe 86.4 million transactions of 1.89 million bond issues, 78% of which occur in the secondary market. The total value of all trades was $17.7 quadrillion (or $1.77 trillon per year), of which 62% occurs in the secondary market. Hence, primary market trades tend to be larger; the average par on transactions in the primary market was $370 thousand while only $160 thousand in the secondary market. The average maturity of issues traded was 17.83 years at an average yield of 3.9%. There is not a significant difference between the yield on trades in the primary market versus the secondary market, 3.97% and 3.90%, respectively. Table 1 reports descriptive statistics.

Using the moments above, we calibrate the model as follows. We assume that the distribution of investors' valuations, F, is a truncated log-normal, with parameters μ_F and σ_F. The lower limit of the truncation, $\underline{v}$, is kept at zero and the upper limit, $\bar{v}$, is set at $\bar{v} = F^{-1}(0.999)$. The issuance cost is $c = \frac{\tilde{c}}{r+\mu}$, where $\tilde{c}$ is also truncated log-normal, with parameters μ_G and σ_G, and the limits are the same as in the F distribution (that is, $\underline{c} = \underline{v}$ and $\bar{c} = \bar{v}$). The above specification gives eight parameters to calibrate:

[14] Alternatively, we could follow the MSRB in denoting all "new issue trades" as those occuring within the first 30 days. However, we chose to be consistent with the literature for comparison.

r, μ, λ_a, λ_b, μ_F, σ_F, μ_G and σ_G. We calibrate to an annual frequency and set the discount rate $r = 0.05$. We set the maturity parameter so that the average maturity in the model matches the one in the data, $\mu = 1.0/17.83$.

This leaves us with six parameters to calibrate, the frequency of trade opportunities in the primary and secondary market, λ_a and λ_b, and the mean and variance of the value and cost distributions. We use the following moments in the data to discipline these 6 parameters. In order to capture the importance of the secondary market for municipal bonds, we the target the relative size of this market in the total value of trade of 62%. This helps discipline the relative size of λ_b versus λ_a. As we discuss in the simple version of the model, the shape of the distribution of valuations and costs are important in understanding the informational rents that arise from the private information. To discipline these distributions, we target the average yield on transactions in the primary and secondary market as well as the average dispersion of yields of trades within an issue, over time. This gives us five moments to match six parameters. For the last moment, we use data on the expected number of times an issue is traded on the secondary market as a function of the amount of time since issuance. In the data, we observe that as issues mature, they are less likely to be traded. The same is true in the model since, on average, issues are allocated from low-valuation investors to high-valuation investors. Hence, the path of the frequency of trade also helps us discipline the distribution of valuations.

Table 2: Calibrated parameters

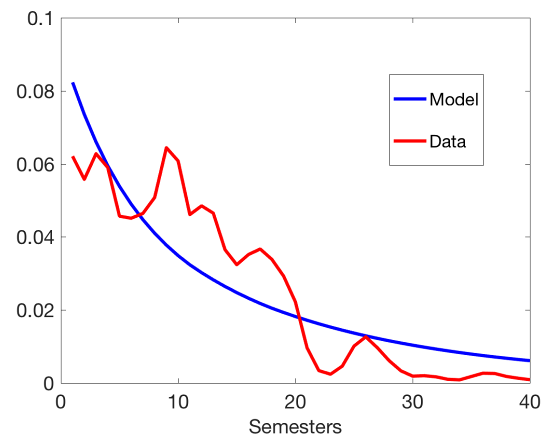
r	μ	λ_a	λ_b	μ_F	σ_F	μ_G	σ_G
0.050	0.056	0.100	4.650	-0.001	0.970	-0.097	0.688

Table 2 depicts the calibrated parameters of the model, while table 3 and figure 2 depict the outcomes of the model in comparison with those in the data. In the general, the model does very well matching the data, with the exception being the average yields in the primary and secondary markets. The reason is that the model always generates prices in the secondary market higher than those in the primary market since investors who sell assets in the secondary market did not change valuation since they originally bought in the primary market. Hence, if they re-sell, it must be at a higher price or lower yield. In the data, while the yield is lower in the secondary market, quantitatively they are very close, so the model cannot match it exactly. It is also worth mentioning that the model matches well the average yield for all transactions. In the data, average yield in both markets is 3.91% while in the model it is 4.00%.

Table 3: Moments

Variables	Data	Model
Average maturity	14.96	14.96
Primary/Total market	0.38	0.39
Primary market		
Average yield	3.96	5.66
Yield std. deviation	0.30	0.31
Secondary market		
Average yield	3.90	2.96
Yield std. deviation	1.10	1.10
Average yield	3.91	4.00

Figure 2: Trade after issuance



5.1 The pattern of trades

In the secondary market, trade occurs when two investors meet, one who owns the asset and one who does not, and the virtual valuation of the non-owner of the asset is above the reservation value of the asset's owner. The intuition for this is the same we discussed in the context of our simple model. Figure 3 depicts the reservation value and the virtual valuation of investors. The virtual valuation is strictly below the reservation value, which implies that trade is inefficient. That is, for some meetings between an investor who owns an asset and one who does not, trade will not occur in some situations where the reservation value of the non-owner of the asset is higher than the one of the owner. Figure 4 illustrates this result. It depicts the region of asset owner valuations, ν_o, and non-owner valuations, ν_n, in which meetings result in trade. For all pairs of valuations (ν_o, ν_n) below the fourth-five degree line, trade is the efficient outcome. However, it only happens in the purple trade region. In the pink region there is not trade.

Trade occurs in the primary market in the same way of the secondary market. In a meeting between an issuer and an investor who does not own an asset, the issuer

Figure 3: Virtual valuation

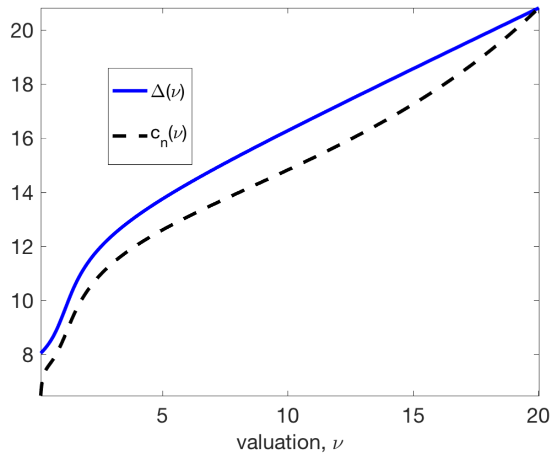
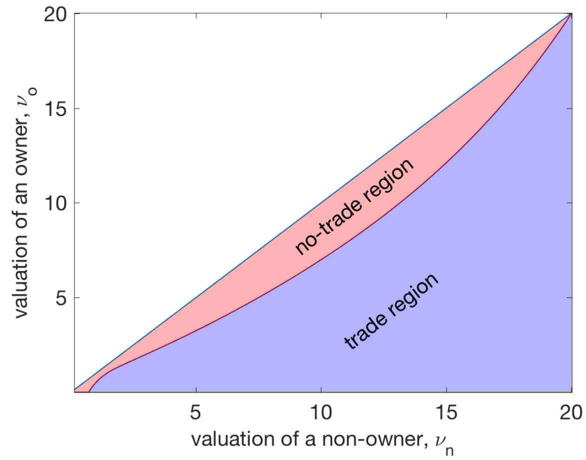


Figure 4: Trade region



issues an asset to the investor if the investor's virtual valuation is above the issuing cost. The fact that trade occurs only when virtual valuations are above the issuance cost, as opposed to needing the reservation value to be above the issuance cost, generates an inefficiency similar to the one in the primary market.

5.2 Asset supply, trade volume, and welfare

There are two important channels in which the private information affects welfare. The first one is a direct channel. Because virtual valuations are lower than reservation values, some meetings do not end up in trade/issuance even though the reservation value of the investor is higher than the reservation value/issuing cost of the potential buyer for the asset. As a consequence, the total surplus in a meeting is not realized. This channel is well explained in our simple version of the model.

The second channel in which private information affects welfare is an indirect channel due to the dynamic nature of asset markets. How much an investor is willing to pay for an asset (the reservation value) depends not only on his own valuation of the asset, ν, but also on the expected surplus from selling the asset, $\pi_o^b(\nu)$, his outside option of buying the asset from another investor, $\pi_n^b(\nu)$, and his outside option of buying the asset from an issuer, $\pi_n^a(\nu)$. The resale value of the asset goes down with private information because the potential buyer can extract informational rents from the investor trying to sell an asset. Moreover, the outside options go up, also because the investor can extract informational rents when buying the asset from another investor or from an issuer. The lower reservation value under incomplete information implies that investors are less willing to pay issuers to issue an asset. As a result, issuance and asset level are both inefficiently low due to private information.

To better understand what is going on in the model it is worth comparing it with a complete information benchmark. The difference between reservation values in the complete and incomplete information models. Which is given by

$$\Delta^{CI}(\nu) - \Delta(\nu) = \frac{\lambda_b[(1-s^{CI})\pi_o^{bCI}(\nu) - (1-s^{CI})\pi_o^{bCI}(\nu)] + \lambda_b s \pi_n^b(\nu) + \lambda_a \pi_n^a(\nu)}{r+\mu} . \quad (35)$$

Equation (35) provides a way to decompose the difference in reservation value in the complete versus the incomplete information model. Figure 5 depicts the difference in reservation values in the calibrated version of the model and Figure 6 decomposes this difference. We see that for low values of ν, the difference is mostly due to losses in resale value. That is because under incomplete information, a low-valuation owner is not able to size the gains from trade associated with meeting a high-valuation non-owner. For higher values of ν the difference is due to the outside option. In this region, if an investor does not own an asset, it is cheap for him to buy in the primary or secondary market and extract informational rents in the process.

The discussed inefficiencies imply that assets are not well allocated because trade in the secondary market is not efficient. Additionally, the total level of assets is not efficient because trade in the primary market is also inefficient. This results in less issuance, leading to a lower asset level, lower trade volume and lower total welfare.

Table 4: Asset level, trade volume and total welfare

	Asset level	Trade volume	Total welfare
Complete information	0.5628	2.6803	24.1911
Incomplete information	0.4365	0.5959	22.3556
Difference	22.44%	77.77%	7.59%

Table 4 illustrates the magnitude of the loss in asset supply, trade volume, and welfare

Figure 5: Reservation value

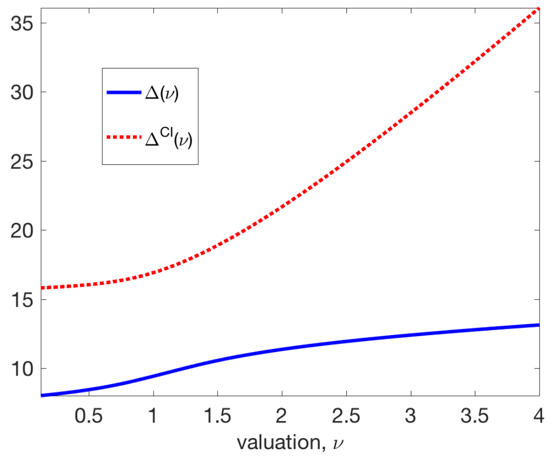
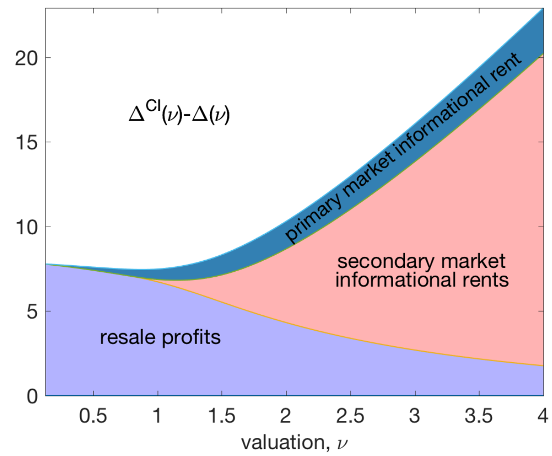


Figure 6: Difference in Δ



for the US municipal bond market. Asset supply is 22.44% lower under incomplete information and secondary market trade volume is 77.77% lower. Note that volume falls more than the level of assets because, besides a reduction in trade opportunities due to less assets in the economy, there are also trades which are lost due to the information friction. In terms of welfare, agents are 7.59% worse off in the incomplete information economy than with complete information.

5.3 Misallocation and the distribution of assets

The inefficiencies generated by the information friction affect the distribution of asset holdings among owners and non-owners. Figure 7 and 8 depicts both these measures in comparison with the analogous measures associated with complete information.

Total assets in the model with incomplete information is $s = 0.4365$, while in our benchmark with complete information it is $s^{CI} = 0.5628$. The difference in total assets, $s^{CI} - s$, is associated with the area between the curves in Figures 7 and 8. The welfare cost associated with complete information is mostly due to investors with intermediate valuations (between 0.5 and 2.5) holding less asset than in the complete information benchmark. Investors with low valuations (ν close to zero) are not likely to hold assets in either economy. Similarly, with high valuation are able to acquire assets in both economies. This pattern reflects a key feature of the trade mechanism, in order to capture

Figure 7: Measure of owners

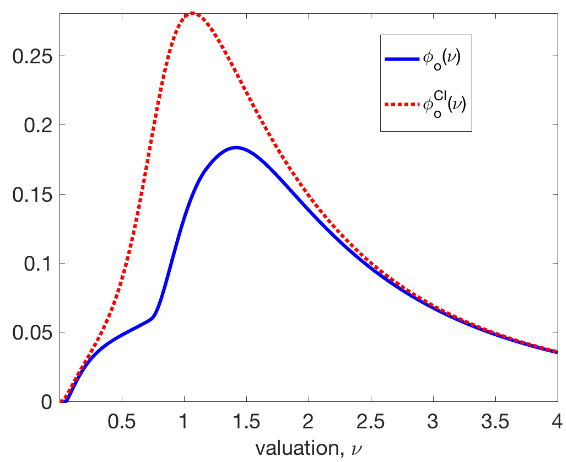
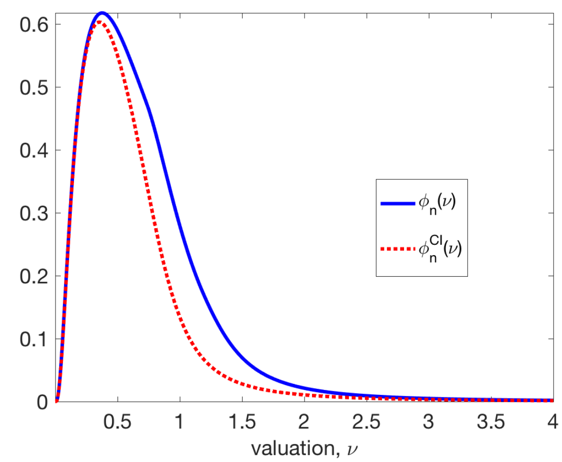


Figure 8: Measure of non-owners



rents from high valuation investors, sellers sacrifice trade with intermediate valuation investors, which generates the pattern seen above.

5.4 Primary *vs* Secondary market

Incomplete information affects trade in both the primary market and the secondary market. A natural question is in which market is the information friction more severe. To answer this question we solve two alternative models. In the first one, there is complete information in all meetings in the primary market, but not in the secondary market. In the second one, there is complete information in all meetings in the secondary market, but not in the primary market. Table 5 repeats the results from Table 4, decomposing the informational effects between the two markets.

Table 5: Decomposition: primary *vs* secondary market

	Asset level	Trade volume	Total welfare	Welfare loss
Incomplete	0.4365	0.5959	22.3556	7.59%
Complete—primary	0.4531	0.6401	22.5844	6.64%
Complete—secondary	0.5312	1.9201	24.0159	0.07%
Complete—both markets	0.5628	2.6803	24.1911	-

The effect of private information in the primary market for municipal bonds is small. In this case, the welfare loss with respect to the complete information benchmark is 6.64% while in the incomplete information version the welfare loss is 7.59%. Solving informational problems in the primary market only increases welfare by 0.95%. On the other hand, private information in the secondary market has a substantial effects. Eliminating private information in secondary market trades nearly implements the complete information allocation. The welfare loss of only having private information in asset issuance is 0.07%.

The fact that the welfare cost of incomplete information comes mostly from the incomplete information in the secondary market may lead some people to think that modelling the primary market is not important, but that is not the case. The reason is that the channel in which the information friction in the secondary market reduces welfare is works through asset supply determined in the primary market. The information friction in the secondary market allow non-owners of assets to extract informational rents from owners. This reduces the value of having an asset (that is, the reservation value). As a result, issuance goes down because the value of buying an asset is lower so investors are less willing to pay for an asset. We can see this by noticing that even when there is complete information only in the primary market, asset supply is still 19.49% below the level under full complete information. However, when there is complete information only in the secondary market, asset supply is only 5.61% below the level under full complete information.

6 Conclusion

References

Bigio, S. (2015). Endogenous liquidity and the business cycle. *American Economic Review*, 105(6):1883–1927.

Chang, B. (2014). Adverse selection and liqudity distortion.

Chari, V., Shourideh, A., and Zetlin-Jones, A. (2014). Reputation and persistence of adverse selection in secondary loan markets. *American Economic Review*, 104(12):4027–4070.

Chiu, J. and Koeppl, T. (2015). Trading dynamics with adverse selection and search: Market freeze, intervention and recovery. *Review of Economic Studies*, 00:1–35.

Cujean, J. and Praz, R. (2015). Asymmetric information and inventory concerns in over-the-counter markets.

Duffie, D., Garleanu, N., and Pedersen, L. H. (2005). Over-the-counter markets. *Econometrica*, 73:1815–1847.

Eisfeldt, A. (2004). Endogenous liquidity in asset markets. *The Journal of Finance*, 59(1):1–30.

Fundenberg, D. and Tirole, J. (1991). *Game Theory*. MIT Press.

Green, R., Hollifield, B., and Schurhoff, N. (2007). Financial intermediation and the costs of trading in an opaque market. *The Review of Financial Studies*, 20(2):275–314.

Guerrieri, V., Shimer, R., and Wright, R. (2010). Adverse selection in competitive search equilibrium. *Econometrica*, 78(6):1823–1862.

Hugonnier, J., Lestera, B., and Weill, P.-O. (2014). Hetergeneity in decentralized asset markets.

Krishna, V. (2010). *Auction Theory*. Academic Press, 2nd edition.

Lagos, R. and Rocheteau, G. (2009). Liquidity in asset markets with search frictions. *Econometrica*, 57:403–426.

Myerson, R. B. (1981). Optimal auction design. *Mathematics of Operations Research*, 6(1):58–73.

Myerson, R. B. and Satterthwaite, M. A. (1983). Efficient mechanisms for bilateral trading. *Journal of Economic Theory*, 29:265–281.

Zhang, S. (2016). Liqudity misallocation in an over-the-counter market.