Monopolizing Violence and Consolidating Power*

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Abstract

Governments in weak states often face an armed opposition and have to decide whether to try to accommodate and contain that adversary or to try to consolidate power and monopolize violence by disarming it. When and why do governments choose to consolidate power and monopolize violence? How fast do they try to consolidate power? When does this lead to costly fighting rather than to efforts to eliminate the opposition by buying it off? We study an infinite-horizon model in which the government in each period decides how much to offer the opposition and the rate at which it tries to consolidate its power. The opposition can accept the offer and thereby accede in the government’s efforts to consolidate, or the opposition can fight in an attempt to disrupt those efforts. In equilibrium, the government always tries to monopolize violence when it has “coercive power” against the opposition where, roughly, coercive power is the ability to weaken the opposition by lowering its payoff to fighting. Whether the government consolidates peacefully or through costly fighting depends on the size of any “contingent spoils” which are benefits that begin to accrue from an increase in economic activity resulting from the monopolization of violence and the higher level of security that comes with it. When contingent spoils are small, the government buys the opposition off and eliminates it as fast as is peacefully possible. When contingent spoils are large, the government tries to monopolize violence by defeating the opposition militarily.

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I. Introduction

Governments in weak states often face one or more armed groups that pose a threat to the government itself or to its control over areas of the country. More than a 100 states faced armed opposition groups between 1950 and 2007. On average, about fifteen percent of states confronted at least one armed group in any given year. These confrontations led to 144 civil wars with the rebels prevailing in about a quarter of them. Governments in poorer countries are even more likely to face armed opposition. On average twenty-two percent of countries with per-capita incomes under $5000 face one or more armed opponents in any one year. These conflicts led to 87 civil wars with the rebels again prevailing about a quarter of time.

A government facing an armed opposition has a choice. It can reconcile itself to facing an armed opponent and attempt to accommodate or contain it. Or, the government can try to consolidate its power and monopolize violence by disarming that faction. If the government chooses the latter course, it can try to disarm the opposition peacefully by buying it off or by defeating it militarily. The opposition too faces a choice. It can accept what the government offers and acquiesce in the government’s efforts to consolidate. Consolidation, however, generally entails a shift in the distribution of power against the opposition and leaves the opposition in a weaker future bargaining position. Alternatively, the opposition can fight to impede the government’s efforts, forestall the adverse shift in power, and possibly defeat the government.

When and why do governments choose to consolidate power and monopolize violence? How fast do they try to consolidate power? When does this lead to costly fighting rather than to efforts to eliminate the opposition by buying it off?

This paper studies these choices in the context of a simple stochastic game in which two factions vie for control of the state and the spoils that come with it. One faction,

1 The data are drawn from the UCDP/PRIO Armed Conflict Dataset.
2 Fearon and Laitin (2007). I am grateful to Jim Fearon for an updated version of the data.
the government, starts out in control of the state. In each of possibly infinitely many periods, the government decides how much to offer the opposing faction, whether to try to consolidate its power, and, if so, how fast to do so. The opposition can accept the offer or fight in an attempt to forestall the government’s efforts to consolidate. Neither faction is able to commit to future actions. The government is unable to commit to future transfers. The opposition is unable to commit to not fighting in the future. The distribution of power between the government and the opposition is the state variable of the game.

The analysis makes three main contributions. The first shows that the government always consolidates power and weakens the opposition whenever the government has “coercive power.” Coercive power is defined precisely below. Roughly, the principal in a principal-agent model has coercive power when, in addition to being able to make an offer to the agent, the principle can also take an action that lowers the agent’s reservation value. In Acemoglu and Wolitsky’s (2011) model of coercive labor relations, for example, the slave owner (principal) can buy guns which lower the slave’s (agent’s) reservation value, i.e., the slave’s payoff to trying to run away. When the government has coercive power, it always uses it to weaken the opposition. When the government lacks coercive power, it must fully compensate the rebels today for their agreeing to be weaker tomorrow and, therefore, may be indifferent to monopolizing violence.

Second, whether the government tries to consolidate peacefully by buying the opposition off or by defeating the opposition militarily depends on the size of the “contingent spoils.” These are benefits which only begin to flow once the government or the opposition (by defeating the government) has monopolized violence. Contingent spoils model the idea that once one faction has monopolized violence, it can provide a level of security and protection conducive to investment and economic growth. Both theory and empirical evidence suggest that growth and investment are inversely related to the threat or
actual use of violence.\textsuperscript{3} Consider then three stylized situations: there is actual fighting between the government and the opposition; the government and opposition have agreed not to fight but both remain armed and capable of fighting; and either the government or the opposition has monopolized violence by eliminating the other. Absent the ability to commit to not using force in the future, there is a latent threat of renewed fighting in the second situation and not in the third.\textsuperscript{4} As a result, the returns to investment are likely to be higher in the latter. These returns are contingent spoils and may include increases in foreign direct investment, domestic investment, the ability to exploit oil or mineral wealth, some forms of development assistance or foreign aid, or more generally the returns from any increase in economic activity resulting from the monopolization of violence and the enhanced security that comes with it. More conceptually, contingent spoils are the gains a state reaps from the additional commitment power it attains from having disarmed an opposing faction rather than only agreeing with that faction to stop fighting.

Contingent spoils create a trade-off. Peaceful consolidation avoids the deadweight losses due to fighting. But it also takes time to buy the opposition off, gradually weaken it, and ultimately eliminate it. Because the government can only commit to divisions of today’s “pie” and not to future transfers, the government faces a liquidity problem. If the government tries to consolidate too quickly, it will be unable to offer the opposition enough today to compensate the opposition for being much weaker tomorrow. Thus, there is an upper limit on how fast the government can consolidate peacefully. This in turn delays the realization of any contingent spoils. If these gains are small, the cost of fighting outweighs the cost of delay and the government consolidates peacefully in equilibrium.

\textsuperscript{3} See, for example, Collier (1999); Abadie and Gardeazabal (2003); Collier, Hoeffler, and Patillo (2004); Cerra and Saxena (2008); and Suliman and Mollick (2009). Blattman and Miguel (2010) provide an overview. If we take violence to be an extreme form of political instability, then Barro (1991), Alesina and Perotti (1996), Rodrik (1999), and Aisen and Viega (2011) all find a negative relationship between political instability and growth.

\textsuperscript{4} Licklider (1995) found that about half of the negotiated civil-war settlements were followed by renewed fighting. Walter (2004) reports that 22 out of 58 civil wars ending between 1948 and 1996 were followed by another war. Toft (2010) finds that civil wars ending in one side’s decisive military defeat are much less likely to see future fighting.
If, by contrast, the contingent spoils are sufficiently large, the cost of delaying these gains outweighs the cost of fighting and equilibrium play entails fighting.

These trade-offs lead to three types of equilibrium path. If the contingent spoils are small, the government monopolizes violence peacefully and eliminates the opposition as rapidly as is peacefully possible. If the contingent spoils are sufficiently large, the government consolidates power by fighting in either the first or second round of the game.

What determines whether fighting occurs in the first or second round is the trade-off between the cost of delaying the contingent spoils for one period versus the gain from fighting on better terms in the second round. When this gain is sufficiently large, there is a one-period “truce” when the factions delay fighting until the second period. The gain derives from two sources. By delaying a fight, the government can exploit its coercive power. The government can also create and capture efficiency gains by shifting the distribution of power in such a way that fighting becomes more decisive and therefore less costly. (If the factions fight in a given round, decisiveness is the probability that one faction achieves a monopoly of violence in that round by defeating the other faction.)

The third main contribution is to endogenize the dynamics of consolidation and shifting power in a setting where there is also an explicit decision to fight. This is important for two reasons. First, as Blattman and Miguel (2010) point out, commitment problems resulting from incomplete contracting is one of the leading theoretical explanations for civil war. These models typically take the shifts in power that create these commitment problems to be exogenous (e.g., Fearon 1998, 2004; Walter 2002; Fearon and Laitin 2008; Chassang and Padro 2009; Powell 2006, 2012). The main result is that large shifts lead to fighting because the government is unable to compensate the rebels enough to day for them to forego fighting and be weaker tomorrow. Fearon and Laitin (2008) also find evidence of an exogenous shock to the distribution of power – typically in the form of a change in foreign support – in two-thirds of their cases of civil war termination.

But many shifts in power and, especially, those related to state consolidation would seem to be endogenous or at least to have a significant endogenous component. For
example, the faction controlling the state often tries to consolidate its position and shift power in its favor by taking over and politicizing the police, army, and internal security forces; arming its own militia; weakening the opposition by arresting, eliminating, or isolating its leaders; and effectively weakening or disenfranchising opposition groups. Al-Maliki has been accused of most of these things in Iraq. Perhaps most brazenly, hours after the U.S. formally withdrew its forces from Iraq, Iraqi troops under the command of Al-Maliki’s son placed Vice President Tariq al-Hashemi, Finance Minister Rafi al-Issawi and Deputy Prime Minister Saleh al-Mutlaq under house arrest. All three were leaders of the major opposition party (Dodge 2012). Al-Hashemi was subsequently sentenced to death in absentia (Al-Jawoshy and Schwirtz 2012).

Studying endogenous shifts with explicit decisions to fight or not is important for a second reason. Existing models of civil and interstate war generally take one of two forms. One type assumes the distribution of power to be exogenous but explicitly models the actors’ decisions to fight (e.g., Fearon 1995, 2004, 2007; Fearon and Laitin 2008; Jackson and Morelli 2007; Leventoğlu and Tarar 2008; Powell 1999, 2006, 2012; and Slantchev 2003). The second type endogenizes the distribution of power, usually as a choice between consuming and arming. But these models typically do not include an explicit decision to fight once the parties have armed (e.g., Besley and Persson 2010, Beviá and Corchón 2010, Dal Bó and Dal Bó 2011). Arming in these models is equivalent to fighting. As a result, these analyses do not address the “inefficiency” puzzle which has framed much recent work on the causes of inter-state and civil war (Fearon 1995, Powell 2006). Given the deadweight cost of fighting, there are agreements that are Pareto superior to fighting. Why do the factions fail to reach one of these agreements and thereby avoid costly fighting? Contingent spoils provide an answer to this question in the context of dynamic endogenous consolidation.6

The next section presents the model. Section III characterizes the equilibria, and Sec-

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tion IV presents some comparative static results. The subsequent three sections describe extensions, related work, and provide some empirical illustrations. Proofs of the main results are in the appendix.

II. A Model

The model formalizes the interaction between a government and an armed opposition in a weakly institutionalized polity where the rule of law is absent or weak. The factions are vying for control of the state and the spoils that come with it. The government must choose whether to try to consolidate its power and, if so, how fast. It must also decide whether to attempt to consolidate peacefully by buying the opposition off or by defeating it militarily. The opposition can either accept the offer and thereby acquiesce in the government’s efforts, or the opposition can fight. The substantive import of the assumption of institutional weakness is that neither the government nor the opposition can commit to future actions. More specifically, neither can commit to how the social “pie” will be divided in the future. Whatever division they may agree to now can be renegotiated in the future in light of any changes in the distribution of power. Moreover, neither the opposition nor the government can commit to not using force in the future.\footnote{See Acemoglu (2003) for a discussion of weakly institutionalized settings and why the Coase Theorem does not apply.}

Formally, consider a two-player, infinite-horizon stochastic game in which the faction in charge of the government, $G$, and an armed rival faction, $R$, are trying to divide a flow of “pies.” The size of the pie to be divided in each period is one as long as both factions remain armed. Should one of the factions ever establish a monopoly of violence by disarming the other faction either by force or by agreement, the per-period flow of benefits increases to $1+\gamma$. The parameter $\gamma \geq 0$ measures the size of the contingent spoils which begin to flow once one faction has obtained a monopoly of violence and is thereby able to provide a high degree of security and protection. Examples of such spoils might include FDI, the ability to exploit oil or mineral wealth, or some forms of development assistance or foreign aid. The factions share a common discount factor $\beta$, each tries to maximize its sum of discounted spoils, and it is convenient to let $V \equiv 1/(1-\beta)$ denote
the present value of the flow of pies of size one.

At the start of any round $t$, $G$ makes a take-it-or-leave-it proposal $\rho_t$ which $R$ can accept or reject by fighting. If $R$ accepts, the proposal is implemented and play moves on to round $t+1$ which starts with a new proposal from $G$. We describe the nature of the proposal below. If $R$ fights, $G$ and $R$ get flow payoffs of $f_G \geq 0$ and $f_R \geq 0$ for that period where $f_G + f_R < 1$ since fighting is inefficient. Fighting also means that the round will end in one of two ways: Either the game ends because one faction decisively defeats the other, or play continues on to the next round because the fighting is inconclusive. If one faction defeats the other, the game ends with the victor getting the entire flow of future spoils and the loser getting nothing. That is, $G$ and $R$ respectively obtain $f_G + \beta (1 + \gamma)V$ and $f_R$ if $G$ prevails and $f_G$ and $f_R + \beta (1 + \gamma)V$ if $R$ prevails.\(^8\)

Let $d_t \in [0, 1]$ be the “decisiveness” of fighting at $t$, i.e., the probability that the game ends if $R$ fights, and take $p_t \in [0, 1]$ to be the conditional probability that $R$ prevails given that the game ends at $t$. The pair $(d_t, p_t)$ defines the distribution of power at time $t$. The distribution of power also defines the state or “stage” of the game with $s_t \equiv (d_t, p_t)$.\(^9\)

$G$’s proposal $\rho_t$ is composed of two parts with $\rho_t \equiv (z_t, \sigma_{t+1})$. The first, $z_t \in [0, 1]$, is the share of the current pie $G$ is offering to $R$. The second component $\sigma_{t+1} = (d, p) \in [0, 1]^2$ is the distribution of power or stage to which play will move if $R$ accepts and thereby acquiesces in the government’s efforts to consolidate its power. We assume $G$ can independently pick any $p_{t+1} \in [0, 1]$ and $d_{t+1} \in [0, 1]$, i.e., $G$ can choose any $\sigma_{t+1} \in [0, 1]^2$.

Including $\sigma_{t+1}$ in the proposal is a simple, reduced-form way of endogenizing $G$’s efforts to consolidate power. We discuss this specification and alternatives below.

Figure 1 illustrates the sequence of play. If $R$ accepts $(z_t, \sigma_{t+1})$, $R$ and $G$ respectively get $z_t$ and $1 - z_t$ during that round, the distribution of power shifts from $s_t \equiv (d_t, p_t)$ to $s_{t+1} \equiv (d_{t+1}, p_{t+1})$.\(^7\)

\(^8\) On substantive grounds, $G$ as well as $R$ should be able to choose to fight. In effect, $G$ can do this by making an offer $R$ is certain to reject. Such offers are sure to exist as shown below. Note, however, that this simplification conflates fights $R$ starts with those $G$ starts. As a result, an attack $R$ launches yields the same distribution over outcomes as an attack $G$ launches.

\(^9\) We use the term “stages” for the states in the stochastic game in order to avoid confusing the state of the game with the bureaucratic state which is consolidating.
and the next round begins in stage $s_{t+1} = \sigma_{t+1}$ with a new offer $(z_{t+1}, \sigma_{t+2})$ from $G$. Fighting impedes $G$’s efforts to consolidate. More specifically, if $R$ fights and the fighting is inconclusive, then the distribution of power remains the same (i.e., $s_{t+1} = s_t = (d_t, p_t)$) with probability $1 - \epsilon$. With probability $\epsilon \geq 0$, $G$’s efforts to consolidate its power succeed despite the fighting and distribution of power in the next round is $s_{t+1} = \sigma_{t+1}$.

To formalize the possibility that $G$ eliminates $R$ through peaceful consolidation rather than military defeat, assume that $R$ has been effectively eliminated as an armed group if play reaches $E \equiv (1, 0)$. Were the factions to fight at this stage, the fighting is sure to be decisive and $R$ is certain to lose. Then $G$ eliminates $R$ peacefully if $R$ agrees to move to $E$. The game ends at this point; contingent spoils begin to flow; and $G$ and $R$ respectively obtain payoffs $(1 + \gamma)V - f_R$ and $f_R$, respectively.\(^{10}\)

This set up is very spare, and it is useful to elaborate on four aspects of it. First, letting $G$ specify the next stage of the consolidation process is a reduced form. It captures in a simple way the notion that when $G$ offers $z_t$, $G$ can also take unmodelled observable actions which if unopposed will shift the distribution of power from $s_t$ to $\sigma_{t+1}$ at $t + 1$. ($G$ could also do nothing which would be represented by $\sigma_{t+1} = s_t$.) As noted

\(^{10}\) Assuming $R$ to get a residual payoff of $f_R$ when it is disarmed keeps the payoffs to fighting to the finish continuous in $d_k$ and $p_k$, and this simplifies the analysis.
above, possible actions include taking over and politicizing the police, army, and internal security forces; arming its own militia; weakening the opposition by arresting, eliminating, or isolating its leaders; and effectively weakening or disenfranchising opposition groups. These efforts also include attempts to “win hearts and minds” (e.g., Crost, Felter, and Johnson 2011; Berman, Shapiro, and Felter 2011). The assumption that these actions are costless simplifies the analysis, and some of the implications of this simplification are discussed below.

Second, representing the probabilities of the three possible outcomes of fighting in terms of its decisiveness, $d_t$, and the conditional probability that $R$ prevails, $p_t$, is just one of many formally equivalent ways of defining the probabilities associated with the outcomes of fighting. Another obvious possibility is in terms of the unconditional probabilities that the government and opposition prevail at time $t$, say $\pi_t^G$ and $\pi_t^R$. Then $\pi_t^R = p_t d_t$, and $\pi_t^G + \pi_t^R = d_t$ (see, for example, Fearon 2004).

The specification used here reflects two considerations. The cost or efficiency loss due to fighting turns out to play a key role in describing the equilibria of the game. These costs depend on the expected duration of a fight, and this is most directly related to decisiveness.

The other consideration is that there is a natural empirical interpretation of $d_t$ and $p_t$. Assume the probabilities of prevailing in a single round remain constant at $\pi_t^G$ and $\pi_t^R$, and consider a fight to the finish in which $G$ and $R$ keep fighting until one of them is defeated. Then the probability that $R$ prevails in this contest is the sum of the probabilities that it prevails in round $j$ for $j \geq 0$ which is $\sum_{j=0}^{\infty} \pi_t^R [1 - (\pi_t^G - \pi_t^R)^j] = \pi_t^R / (\pi_t^G + \pi_t^R) = p_t$. The probability that $G$ prevails is $1 - p_t$, and the expected duration of the fight is $1/d_t$. Thus anything that affects the likelihood that $G$ or $R$ ultimately prevails in a fight to the finish works through $p_t$ whereas anything that affects the expected duration works through $d_t$.

The third aspect of the model that requires elaboration is the assumption that $G$ can independently choose any $p_{t+1} \in [0, 1]$ and any $d_{t+1} \in [0, 1]$ when making proposal $\sigma_{t+1}$. This is clearly a simplifying assumption. To develop it a bit further and provide some empirical referents for the actions that a government could actually take, consider a still
simpler assumption. In many conflicts, the government has an overwhelming military advantage against the insurgents. The problem is finding them. This was a key aspect of the insurgency in Iraq. When one tribal leader turned against Al-Qaida as part of the Sunni Awakening, he personally reported 130 members of Al-Qaida from his tribe (Cigar 2011, 44). The problem of finding insurgents is also a central premise of the U.S. Army’s counter-insurgency strategy (U.S. Army 2007) as well as the point of departure for Fearon’s (2008) and Berman, Shapiro and Felter’s (2011) analyses.

If we think of $\delta$ as the probability of finding the guerrilla leaders in period $t$, then a natural formalization of this situation would be that, conditional on finding the rebel leaders, the government is virtually certain to win, i.e., $p_t = 0$ for all $t$. Anything that increases the government’s prospects of finding the rebels increases $\delta$. Such measures include investing more in intelligence, e.g., signals intelligence in order to tap into the rebels’ communications and trace their location. Or the government might attempt to buy the hearts and minds of local villagers who can in turn identify the insurgents. Under this assumption, $G$’s proposal at time $t$ is defined by its offer, $z_t$, and by how decisive fighting will become, $d_{t+1}$, with $p_s = 0$ for all $s$.

Relaxing this assumption, the government in some situations must not only find the rebels, but it must also be able to bring to bear enough of its power to defeat them. This is unlikely to be major concern in urban settings or other situations where significant government forces are nearby. It will be more of an issue in some rural settings. So, for example, the government of Colombia secured U.S. funding under “Plan Colombia” for helicopters which “provided the air mobility needed to rapidly move Colombian counter-narcotics and counterinsurgency forces” (GAO 2008). Measures like this would seem to increase the probability of defeating the rebels conditional on having found them, i.e., lowering $p_t$.

After describing the equilibria when the set of feasible future proposals at $t$ includes any $\sigma_{t+1} \in [0, 1]^2$, we discuss the implications of two alternative assumptions. The first is that the rebels cannot defeat the government but only impose costs on it. That is, $p_t = 0$ for all $t$ and $G$ chooses $d_{t+1}$ at $t$. At the other extreme, we fix $d_t = d$ for all $t$. This means
that there are no efficiency gains due to increasing the decisiveness of fighting, and $G$ chooses $p_{t+1}$ when making a proposal at $t$.

Finally, the model assumes the per-period flow of benefits discontinuously increases from 1 to $1+\gamma$ if $R$ agrees to $E$. This is a simple way of modeling the idea that economic activity and the per-period spoils increase when the rebels are sufficiently weak and no longer pose a serious threat. At the cost of much additional complexity, one could assume instead that the per-period spoils rise smoothly from 1 to $1+\gamma$ as $s_k$ approaches $E$.

### III. Paths to Consolidation

This section characterizes the equilibrium paths and payoffs of the pure-strategy, Markov Perfect equilibria (MPE).\(^{11}\) If the government lacks coercive power and if there are no contingent spoils ($\gamma = 0$), then the government is indifferent to disarming the opposition or living with an armed adversary. From the government’s perspective, the amount it has to offer the opposition today in order to compensate it for agreeing to be weaker tomorrow just offsets the value of being able to exploit the weaker adversary tomorrow. This changes when the government has coercive power. The government always tries to monopolize violence when it has coercive power even if there are no contingent spoils. The government buys the opposition off and eliminates it as fast as is peacefully possible, i.e., induces $R$ to move to $E$ as fast as is peacefully possible, when the contingent spoils are small or absent. The government tries to eliminate the opposition by defeating it militarily when the contingent spoils are sufficiently large.

The details of the proofs are quite cumbersome, and we focus here on the main intuitions. Let $\mathcal{E}$ be a pure-strategy MPE and take $V_j(s_k)$ to be $j$’s continuation payoff starting from $s_k$ for $j \in \{G, R\}$.\(^{12}\) Now consider $G$’s decision at any $s_k$. Either $G$ tries to consolidate by fighting (i.e., by making an offer $R$ is sure to reject), or $G$ buys $R$ off.

Suppose $G$ decides to buy $R$ off at $s_k$ by offering $(z_k, s_{k+1})$ where this may or may not

\(^{11}\) See the appendix for a formal description of the strategies and the Markov restriction.

\(^{12}\) We restrict attention to pure strategies. Existence is assured by construction in Proposition 1A.
be $G$’s equilibrium offer at $s_k$.\footnote{Abusing notation to simplify the exposition, we write proposals as $(\zeta_k, s_{k+1})$ rather than $(\zeta_k, \sigma_{k+1})$.} Regardless, $R$ accepts whenever the proposal satisfies $R$’s “peaceful participation constraint” at $s_k$:

$$z_k + \beta V_R(s_{k+1}) \geq f_R + \beta d_k p_k (1 + \gamma) V + \beta (1 - d_k) [(1 - \varepsilon) V_R(s_k) + \varepsilon V_R(s_{k+1})].$$

Even if $(z_k, s_{k+1})$ is an out-of-equilibrium proposal, $R$ expects $G$ in the MPE to revert to its equilibrium strategy in subsequent play. Accordingly, the left side of PPC is $R$’s payoff to accepting, and the right side is its payoff to fighting.

The key to analyzing the dynamics of consolidation is to observe that as long as $d_k < 1$ and $\varepsilon > 0$, $G$ can take actions which affect $R$’s reservation value. More specifically, the coefficient of $V_R(s_{k+1})$ on the right side of PPC is positive. Accordingly, $G$ can relax $R$’s peaceful participation constraint by making proposals $(z_k, s_{k+1})$ which weaken $R$, i.e., have smaller values of $V_R(s_{k+1})$. When $G$ can affect $R$’s reservation value, $G$ has coercive power.

**Definition 1:** $G$ has coercive power at $s_k$ when $d_k < 1$ and $\varepsilon > 0$.

If $G$ buys $R$ off at $s_k$, it does so by consolidating its power: If $G$ can monopolize violence by inducing $R$ to move to $E$ with a single offer (i.e., if PPC is satisfied if $G$ offers $(1, E)$), then $G$ does so. Otherwise, $G$ weakens $R$ as much as possible by minimizing $V_R(s_{k+1})$ subject to satisfying PPC and $z_k \leq 1$.

A two-part intuition underlies $G$’s consolidation of power. Insofar as $G$ makes all of the offers, it has all of the bargaining power, and we would expect $G$ to hold $R$ down to its reservation value in equilibrium. That is, PPC binds whenever $G$ buys $R$ off at $s_k$. (See the proof of Proposition 1A in the appendix. Trivially, PPC must bind if $z > 0$; otherwise $G$ could profitably deviate to $(z_k', s_{k+1})$ for a $z_k' < z_k$.)

In addition to exploiting its bargaining power by holding $R$ down to its reservation value, $G$ can profitably exploit its coercive power and this results in $G$’s consolidation of power. To trace the logic, observe that PPC will generally bind at (infinitely) many proposals. At each of these, $R$ is indifferent between fighting and accepting. $R$, however,
is not indifferent among them when $G$ has coercive power. Suppose, for example, that
PPC binds at $(z'_{k+1}, s'_{k+1})$ and $(\tilde{z}_k, \tilde{s}_{k+1})$ with $V_R(s'_{k+1}) > V_R(\tilde{s}_{k+1})$. Then $R$’s payoff to
accepting equals its payoff to fighting. The latter is independent of $z_k$ and increasing in $V_R(s_{k+1})$. $R$, therefore, prefers $(z'_{k+1}, s'_{k+1})$ to $(\tilde{z}_k, \tilde{s}_{k+1})$. If, by contrast, $G$ lacks coercive
power, $R$’s reservation value is independent of $V_R(s_{k+1})$, and $R$ would be indifferent
between $(z'_{k+1}, s'_{k+1})$ and $(\tilde{z}_k, \tilde{s}_{k+1})$. In symbols, $z'_{k+1} + \beta V_R(s'_{k+1}) = \tilde{z}_k + \beta V_R(\tilde{s}_{k+1})$.

As might be expected, $G$ has the opposite preferences when buying $R$ off. More for
$R$ at $s_k$ means less for $G$ conditional on not fighting and therefore not incurring any
deadweight loss at $s_k$. $G$ therefore prefers $(\tilde{z}_k, \tilde{s}_{k+1})$ to $(z'_{k+1}, s'_{k+1})$. In effect, then, $G$ wants
to maximize $R$’s payoff $z_k + \beta V_R(s_{k+1})$ subject to PPC, $z_k \in [0,1]$, and $V_R(s_{k+1}) \geq f_R$.
(Since $R$ can always obtain at least $f_R$ by fighting, $V_R(s_j) \geq f_R$ for all $s_j$.)

To see what this implies about $G$’s offer, assume PPC binds and rewrite it as $z_k + \beta[1−\varepsilon(1−d_k)]V_R(s_{k+1}) = f_R + \beta d_k p_k (1 + \gamma)V + \beta(1−d_k)(1−\varepsilon)V_R(s_k)$. The expression on the
right is independent of $G$’s proposal. It follows that there is an inverse relation between
$z_k$ and $V_R(s_{k+1})$. The larger $z_k$, the smaller $V_R(s_{k+1})$. This formalizes the tradeoff facing
$G$ when it tries to buy $R$ off. $G$ must offer more today (a higher $z_k$) in order to get $R$ to
agree to being weaker tomorrow (a lower $V_R(s_{k+1}))$.

Given this tradeoff, $G$ reduces $R$’s payoff (and increases its own) by using $z_k$ to buy
down $V_R(s_{k+1})$. If $G$ can induce $R$ to move to $E$ with a single offer, it does so. This
avoids both the cost of fighting and any delay of the contingent spoils. More formally,
if the proposal $(1, E)$ strictly satisfies PPC at $s_k$, then $G$ offers just enough to induce
$R$ to agree to $s_{k+1} = E$ where $V_R(s_{k+1}) = V_R(E) = f_R$. $G$ in other words monopolizes
violence by eliminating $R$ when it can do so with a single offer. If $R$ is too strong and
will not agree to $E$ in return for a $z_k < 1$, then $G$ weakens $R$ as much as possible by
offering $z_k = 1$.

\footnote{The contingent spoils begin to flow in the next round, i.e., at $s_{k+1} = E$, if $G$ buys $R$
off with a single offer. If $G$ were to fight at $s_k$, the contingent spoils could not begin to
flow any sooner.}

\footnote{As shown in the appendix (see ??), $(1, E)$ strictly satisfies PPC when $[\beta + \beta d_k p_k (1 + \gamma)V + \beta \varepsilon(1−d_k)f_R]/[1 − \beta(1−d_k)(1−\varepsilon)] < 1 + \beta f_R$.}
Proposition 1 summarizes these results.

**Proposition 1 (Consolidating Power):** If $G$ has coercive power at $s_k$ and the factions do not fight at $s_k$ in $E$, then $G$ (generically) consolidates power at $s_k$: If $G$ can induce $R$ to accept $E$ at $s_k$, it does so at the minimal $z_k$ satisfying PPC given $s_{k+1} = E$. Otherwise, $G$ offers $z_k = 1$ in return for weakening $R$ as much as possible by moving to an $s_{k+1}$ which minimizes $V_R(s_{k+1})$ subject to PPC.\(^{16}\)

Proposition 1 shows that $G$ consolidates power at $s_k$ when it has coercive power regardless of whether there are any contingent spoils (i.e., for all $\gamma \geq 0$). It will be useful in what follows to describe $G$’s actions when $d_k = 1$ and consequently $G$ lacks coercive power. This will also help highlight the role that contingent spoils play in consolidation.

Suppose $G$ buys $R$ off in equilibrium at $s_k = (1, p_k)$. Then PPC reduces to $z_k + \beta V_R(s_{k+1}) \geq f_R + \beta d_k p_k (1 + \gamma) V$. To develop some intuition, assume $G$ induces $R$ to move to $E$ with $n$ offers. Then $V_G(s_k) = 1 - z_k + \beta (1 - z_{k+1}) + \cdots + \beta^{n-1} (1 - z_{k+n-1}) + \beta^n [(1 + \gamma) V - f_R]$. Using $V_R(s_{k+n}) = V_R(E) = f_R$ and $V_R(s_{k+j}) = z_{k+j} + \beta V_R(s_{k+j+1})$ for $0 \leq j \leq n - 1$, we can rewrite $G$’s payoff as $V_G(s_k) = (1 + \beta^n \gamma) V - [z_k + \beta V_R(s_{k+1})]$. The first term on the right side of the previous equation is the total amount to be divided between $G$ and $R$ given that the contingent spoils begin to flow after $n$ offers. The second term is the total amount $G$ has to transfer to $R$.

Clearly $G$ benefits by being able to use its bargaining power to hold $R$ down to its reservation value and making PPC bind. When it does, $V_G(s_k) = (1 + \beta^n \gamma) V - [f_R + \beta d_k p_k (1 + \gamma) V]$. If there are no contingent spoils ($\gamma = 0$), $G$’s payoff reduces to $V_G(s_k) = V(1 - f_R - \beta d_k p_k)$. In these circumstances, $G$ is indifferent to any $(z'_k, s'_{k+1})$ and $(\tilde{z}_k, \tilde{s}_{k+1})$ at which PPC binds. $G$ no has incentive to offer a higher $z_k$ in order to buy a lower $V_R(s_{k+1})$ and therefore no incentive to consolidate power. If, by contrast, there are any contingent spoils ($\gamma > 0$), $V_G(s_k)$ is decreasing in $n$, i.e., in how long it takes to move $R$ to $E$. This gives $G$ an incentive to weaken $R$ as much as possible.

If the proposal $(1, E)$ strictly satisfies the PPC at $s_k$ (i.e., if $1 + \beta f_R > f_R + \beta p_k (1 + \gamma) V$), then $G$ offers $z_k = \beta d_k p_k (1 + \gamma) V - (1 - \beta) f_R$ which is just enough to induce $R$ to agree to agree to $E$. If $R$ is too strong and will not agree to $E$ in return for a $z_k < 1$, then $G$.

---

\(^{16}\) Play can be more complicated at a set of stages of measure zero. See footnote XXX for a discussion as well as Lemma 2A in the appendix.
weakens $R$ as much as possible by offering $z_k = 1$ in order to realize the contingent spoils as soon as possible. This leaves:

**Corollary 1:** Assume $\gamma > 0$. If $G$ buys $R$ off at $s_k = (1, p_k)$ in $\mathcal{E}$, then $G$ generically offers $z_k = \min\{1, \beta p_k (1 + \gamma) V - (1 - \beta) f_R\}$.

In sum, if $G$ buys $R$ off at $s_k$ in any MPE $\mathcal{E}$, then $G$ consolidates power whenever it has coercive power whether or not there are any contingent spoils (Proposition 1). If $G$ lacks coercive power, it still consolidates power when there are contingent spoils (Corollary 1). By contrast, $G$ has no incentive to consolidate power when it lacks coercive power and there are no contingent spoils.\(^{17}\)

Suppose now that the factions fight at $s_k$ in equilibrium. Then $G$ is no longer constrained by PPC and will try to shift the distribution of power in its favor as much as possible by setting $s_{k+1} = E$. More formally, the probability that play remains at $s_k$, $(1 - d_k) (1 - \varepsilon)$, and the probability that play moves to $s_{k+1}$, $\varepsilon (1 - d_k)$, are independent of what $G$ names as the next stage. As a result, $G$ maximizes its payoff to fighting at $s_k$ when $\varepsilon (1 - d_k) > 0$ by choosing the $s_{k+1}$ which maximizes $V_G(s_{k+1})$. This is $E$.

**Proposition 2:** If $G$ has coercive power at $s_k$ and the factions fight at $s_k$ in $\mathcal{E}$, then $G$ names $s_{k+1} = E$ at $s_k$.

**Proof:** Since $G$ fights at $s_k$, its continuation value satisfies $V_G(s_k) = f_G + \beta d_k (1 - p_k) (1 + \gamma) V + \beta (1 - d_k) [(1 - \varepsilon) V_G(s_k) + \varepsilon V_G(s_{k+1})]$. $V_G(s_k)$ is clearly increasing in $V_G(s_{k+1})$. It therefore attains its unique maximum at $s_{k+1} = E$ if $V_G(s_{k+1}) < V_G(E)$ for all $s_{k+1} \neq E$. Since $V_G(E) = (1 + \gamma) V - f_R$ and $V_R(s_{k+1}) \geq f_R$, it suffices to show $V_G(s_{k+1}) + V_R(s_{k+1}) < (1 + \gamma) V$ for $s_{k+1} \neq E$. But, the maximal flow of benefits starting from $s_{k+1} \neq E$ is $1 + \beta (1 + \gamma) V = V + \beta \gamma V$ since the soonest the contingent spoils can begin to flow is in the next period. Hence, $V_G(s_{k+1}) + V_R(s_{k+1}) \leq V + \beta \gamma V$. Finally, it is trivial to verify that the proposal $(0, E)$ violates PPC and therefore is sure to be rejected. \(\square\)

The previous discussion has focused on play at $s_k$. We now consider the equilibrium paths starting from $s_k$. There are three possible types: the factions fight at $s_k$, the factions

\(^{17}\) When $d_k = 1$ and $\gamma = 0$, $R$’s payoff to fighting at $s_k$ is $f_R + \beta p_k V$. $G$, moreover, is indifferent between living with $R$ at $s_k$, i.e., offering $(z_{k+j}, s_k)$ with $z_{k+j} = (1 - \beta) [f_R + \beta p_k V]$ for all $j \geq 0$, and eliminating $R$. That is, there are multiple equilibrium paths starting from $s_k$. $G$ eliminates $R$ along some but not all of these paths.
fight farther down the equilibrium path, or they never fight and the equilibrium path is peaceful. We specify the payoffs associated with these paths, some of their properties, and when each obtains. To ease the exposition, we assume there are some contingent spoils ($\gamma > 0$).

If the factions fight at $s_k$ in equilibrium, it is a fight to the finish. With probability $d_k$ one faction defeats the other. With probability $\varepsilon (1 - d_k)$, the next stage is $s_{k+1} = E$ and the game ends. With probability $(1 - \varepsilon)(1 - d_k)$, fighting stops the government’s effort to shift the distribution of power to $E$; play remains at stage $s_k$; and $G$ once again fights and names $E$ as the next stage as $G$ always takes the same action in the same stage in an MPE.

The equilibrium payoffs to fighting at $s_k$ follow immediately. Let $F_G(s_k)$ and $F_R(s_k)$ denote these payoffs. Recalling that $V_G(E) = (1 + \gamma)V - f_R$, $F_G(s_k)$ satisfies the recursive relation

$$F_G(s_k) = f_G + \beta d_k (1 - p_k) + \beta (1 - d_k) [(1 - \varepsilon)F_G(s_k) + \varepsilon [(1 + \gamma)V - f_R]].$$

Using $V_G(E) = (1 + \gamma)V - f_R$ and solving for $F_G(s_k)$ gives

$$F_G(s_k) = \frac{f_G + \beta d_k (1 - p_k)(1 + \gamma)V + \beta \varepsilon (1 - d_k)[(1 + \gamma)V - f_R]}{1 - \beta (1 - d_k)(1 - \varepsilon)}.$$

Similarly, $R$’s payoff to fighting at $s_k$ is

$$F_R(s_k) = \frac{f_R + \beta d_k p_k (1 + \gamma)V + \beta \varepsilon (1 - d_k)f_R}{1 - \beta (1 - d_k)(1 - \varepsilon)}.$$

Now suppose that the continuation game starting from $s_k$ is peaceful, i.e., the factions never fight along the equilibrium path. An immediate consequence of Proposition 1 and Corollary 1 is that $G$ eliminates $R$ by inducing it to move to $E$ as fast as is peacefully possible.

To establish this, note that if $G$ can induce $R$ to move to $E$ with a single offer, then Corollary 1 implies that $G$ does so. If $G$ cannot eliminate $R$ with a single round, then Proposition 1 means that $G$ offers $z_k = 1$ in return for moving to $s_{k+1} \neq E$ which minimizes $V_R(s_{k+1})$ subject to satisfying PPC. If $G$ cannot induce $R$ to move from $s_{k+1}$ to $E$ in a single offer, it offers $z_{k+1} = 1$ in return for moving $s_{k+2}$ which minimizes $V_R(s_k)$ subject to PPC, and so on.
This sequence of offers is sure to end in $R$’s agreeing to $E$ in finitely many rounds. If not, then $V_G(s_k) = \sum_{j=0}^{\infty} \beta^j (1 - z_{k+j}) = 0$. $G$, however, can always obtain at least $f_G$ by fighting at $s_k$. So $V_G(s_k) \geq f_G$, and this contradiction ensures that $R$ must agree to $E$ in finitely many rounds if the continuation game is peaceful.

The factions’ payoffs to a peaceful continuation game now follow. If $G$ can monopolize violence with a single offer, it does so by offering $(z_k, E)$ where PPC binds at $z_k$. This leaves $V_R(s_k) = z_k + \beta V_R(E)$. Using $V_R(E) = f_R$ and solving the binding PPC for $V_R(s_k)$ gives $V_R(s_k) = F_R(s_k)$. If $G$ cannot monopolize violence in a single round, it offers $z_k = 1$. Using $V_R(s_k) = 1 + \beta V_R(s_{k+1})$ and solving the binding PPC now gives

\[
V_R(s_k) = B(s_k) \equiv \frac{f_R + \beta d_k p_k (1 + \gamma) V - \varepsilon (1 - d_k)}{1 - \beta (1 - d_k)(1 - \varepsilon) - \varepsilon (1 - d_k)}.
\]

The appendix shows that $G$ can induce $R$ to agree to $s_{k+1} = E$ when $F_R(s_k) < 1 + \beta f_R$ and is unable to do so when $F_R(s_k) > 1 + \beta f_R$ (see Proposition 1A). Roughly, $R$ will agree to move to $E$ when it is very weak, that is, when its payoff in a fight to the finish is very low ($(d_k, p_k)$ is close to $\langle 1,0 \rangle$). Algebra shows that $B(s_k) \geq F_R(s_k)$ if and only if $F_R(s_k) \leq 1 + \beta f_R$. As a result, we can write $R$’s equilibrium payoff more compactly as $V_R(s_k) = \Pi_R(s_k) \equiv \max\{B(s_k), F_R(s_k)\}$.

As for $G$’s payoff, $G$ must transfer $V_R(s_k) = \Pi_R(s_k)$ to $R$ to induce $R$ to move to $E$. Recalling that $V_R(E) = f_R$, let $q(s_k)$ be the smallest integer $m$ such that $\Pi_R(s_k) \leq (1 - \beta^m) V + \beta^m f_R$ where the expression on the right is the payoff to getting one for $m$ periods and then a final payoff of $f_R$. It takes $q(s_k)$ rounds to move $R$ from $s_k$ to $E$ at which point the contingent spoils begin to flow. Hence $G$’s payoff to buying $R$ off in a peaceful continuation game is $V_G(s_k) = \Pi_G(s_k) \equiv (1 + \beta^{q(s_k)\gamma}) V - \Pi_R(s_k)$. For analytic convenience, set $q(s_k) = \infty$ when $\Pi_R(s_k) \geq V$.

Finally, suppose $G$ buys $R$ off at $s_k$ and the factions fight farther down the equilibrium path at $s_{k+n}$. $G$’s incentives to postpone fighting at $s_k$ arise from two sources: $G$’s coercive power and the potential efficiency gains of fighting when it is more decisive and therefore
less costly. To identify these incentives, write $G$’s equilibrium payoff at $s_k$ as

$$V_G(s_k) = 1 - z_k + \cdots + \beta^{n-1}(1 - z_{k+n-1}) + \beta^n F_G(s_{k+n})$$

$$= 1 - z_k + \cdots + \beta^{n-1}[1 - z_{k+n-1} - \beta F_R(s_{k+n})] + \beta^n [F_G(s_{k+n}) + F_R(s_{k+n})]$$

$$= (1 - \beta^n)V - [z_k + \beta V_R(s_{k+1})] + \beta^n [F_G(s_{k+n}) + F_R(s_{k+n})]$$  \hspace{1cm} (1)

where the last line follows by using $V_R(s_{k+n-1}) = z_k + \beta F_R(s_{k+n})$ and $V_R(s_{k+j}) = z_{k+j} + \beta V_R(s_{k+j+1})$ for $j + 1 \leq n - 1$.

The third term in Eq (1) is the (discounted) joint payoff to fighting and reflects the potential efficiency gain from fighting on more decisive terms. Recall that once the factions start fighting, it is a fight to the finish. As a result, the more decisive fighting is when it starts, i.e., the larger $d_{k+n}$, the shorter the expected duration and the lower the expected cost. Since $G$ holds $R$ down it its reservation value, $R$ is indifferent between fighting and not. $R$’s indiffrence means that whatever is saved by fighting when it is more decisive must be going to $G$. In symbols, the joint payoff to fighting $F_G(s_{k+n}) + F_R(s_{k+n}) = [f_G + f_R + \beta d_{k+n}(1 + \gamma)V + \beta \varepsilon(1 + \gamma)V]/[1 - \beta(1 - d_{k+n})(1 - \varepsilon)]$ is independent of the factions’ relative power $p_{k+n}$ and increasing in $d_{k+n}$.

Some sort of efficiency gain appears to underlie many truces and ceasefires. That is, one or both parties frequently believes that the other side is using the respite from fighting to rearm and regroup. Examples include Hamas in Gaza (Barzak 2008, Dunn 2003), Hezbollah in Lebanon (Teslik 2006), Israeli forces in the 1948 Arab-Israeli war (Oren 2002, 5), the Lords Resistance Army in Uganda (Crilly 2008), and the Tamil Tigers in Sri Lanka (Ramesh 2007) to name just a few. Despite the belief that the other is benefiting from the truce, each side continues to abide by it at least for awhile.

The second term in Eq (1) reflects $G$’s coercive power. By deciding not to fight at $s_k$, $G$ can use its coercive power to take a larger share of the total benefits to fighting at $s_{k+n}$, i.e., take a larger share of $V_G(s_k) + V_R(s_k) = (1 - \beta^n)V + \beta^n [F_G(s_{k+n}) + F_R(s_{k+n})]$. More formally, $z_k + \beta V_R(s_{k+1})$ is increasing in $V_R(s_{k+1})$ (assuming PPC binds). $G$ therefore can increase its payoff to fighting at $s_{k+n}$ by weakening $R$ as much as possible at $s_k$ by minimizing $V_R(s_{k+1})$.  

18
G can fully realize the gains from delaying a fighting in a single period and therefore will either fight at \( s_k \) or \( s_{k+1} \) whenever the continuation game at \( s_k \) entails fighting. That \( G \) can realize these gains in one period follows from Lemma 3A which shows that if \( G \) buys \( R \) off in equilibrium, then \( G \) can induce \( R \) to move to a stage where fighting is completely decisive.\(^{18}\) That is, \( R \) is sure to accept a possibly out-of-equilibrium proposal \((z'_k, s'_{k+1})\) with \( s'_{k+1} = (1, p'_{k+1})\). As a result, \( G \) maximizes its payoff to postponing a fight by offering the proposal \((z_k, s'_{k+1})\) and then fighting at \( \tilde{s}_{k+1} \) where \( z_k = 1, \tilde{s}_{k+1} = (1, \tilde{p}_{k+1}) \), and PPC binds at \( \tilde{s}_{k+1} \). Fighting in the next period (i.e., \( n = 1 \)) with \( d_{k+1} = 1 \) maximizes the efficiency gain in Eq (1).\(^{19}\) Offering \( z_k = 1 \) and holding \( R \) down to its reservation value maximizes \( G \)'s coercive gain (by minimizing \( V_R(s_{k+1}) \)).

In effect, there is a one-period truce or ceasefire at \( s_k \). Both factions can fight at \( s_k \). Both know they will fight at \( \tilde{s}_{k+1} \). Yet both agree not to fight at \( s_k \) even though both know that \( G \) is using the time to shift the distribution of power in its favor and fight on better terms.\(^{20}\)

To determine the factions’ payoffs to fighting at \( \tilde{s}_{k+1} \), observe first that \( V_R(s_k) = 1 + \beta V_R(s_{k+1}) \) since \( G \) buys \( R \) off at \( s_k \). Using this and solving the binding PPC with \( z_k = 1 \) gives \( V_R(s_k) = \Pi_R(s_k) \). (If \( G \) can buy \( R \) off and move to \( E \) with a \( z_k < 1, G \) would never fight in equilibrium.) \( G \)'s payoff to offering \((1, (1, \tilde{p}_{k+1}))\) and then fighting at \( \tilde{s}_{k+1} \) is \( 1 - z_k + \beta F_G(\tilde{s}_{k+1}) = \beta F_G(\tilde{s}_{k+1}) \). To pin down \( \tilde{p}_{k+1} \), we can solve the binding PPC for \( \tilde{p}_{k+1} \) using \( V_R(\tilde{s}_{k+1}) = F_R(\tilde{s}_{k+1}) = f_R + \beta \tilde{p}_{k+1}(1 + \gamma)V \).

---

18 This is a consequence of the simplifying assumption that \( G \) can costlessly move to any \( s_{k+1} \in [0.1] \) if \( R \) does not fight. If it were costly to move or if there were capacity constraints, e.g., \( ||s_k - s_{k+1}|| < c \) for some exogenous \( c \), then longer truces might be possible.

19 This assumes that the contingent gains are sufficiently large that \( F_G(\tilde{s}_{k+1}) + F_R(\tilde{s}_{k+1}) \) is \( V \Rightarrow \beta \gamma > (1 - \beta)(1 - f_G - f_R) \). Were this not the case, \( G \) would prefer to eliminate \( R \) peacefully starting from \( s_k \).

20 The coercive gains are generally not enough by themselves to induce \( G \) to postpone fighting. Suppose, for example, that \( G \) were unable to affect the decisiveness of fighting. Assume, that is, that decisiveness is an exogenous parameter \( d \) and that all proposals save for \( E \) must be of the form \((z_k, (d, p_{k+1}))\). Then \( d_k = d_{k+1} = d \) and there are no efficiency gains from fighting later: \( F_G(s_k) + F_R(s_k) = F_G(s_{k+1}) + F_R(s_{k+1}) \). In these circumstances \( G \) prefers fighting at \( s_k \) to moving to \( \tilde{s}_{k+1} = (d, \tilde{p}_{k+1}) \) and fighting at \( \tilde{s}_{k+1} \) where \( \tilde{p}_{k+1} \) satisfies \( V_R(s_k) = 1 + \beta F_R(\tilde{s}_k) \).
The type of equilibrium path from \(s_k\) is determined by \(\max\{\Pi_G(s_k), F_G(s_k), \beta F_G(\bar{s}_{k+1})\}\), and Proposition 3 summarizes the results.

**Proposition 3:** If \(G\) has coercive power at \(s_k\), the equilibrium continuation paths and payoffs in \(E\) are generically determined by \(\max\{\Pi_G(s_k), F_G(s_k), \beta F_G(\bar{s}_{k+1})\}\). The factions fight at \(s_k\) with \(s_{k+1} = E\), \(V_R(s_k) = F_R(s_k)\) and \(V_G(s_k) = F_G(s_k)\) when \(F_G(s_k) > \max\{\Pi_G(s_k), \beta F_G(\bar{s}_{k+1})\}\). The continuation game is peaceful with \(V_R(s_k) = \Pi_R(s_k)\) and \(V_G(s_k) = \Pi_G(s_k)\) when \(\Pi_G(s_k) > \max\{F_G(s_k), \beta F_G(\bar{s}_{k+1})\}\). The factions fight at \(\bar{s}_{k+1}\) with \(V_R(s_k) = \Pi_R(s_k)\) and \(V_G(s_k) = \beta F_G(\bar{s}_{k+1})\) when \(\beta F_G(\bar{s}_{k+1}) > \max\{\Pi_G(s_k), F_G(s_k)\}\).

In sum, when a government has coercive power, it uses it to weaken the opposition and eventually monopolize violence. Commitment problems created by large shifts in the distribution of power limit the rate at which peaceful consolidation can occur and thus delay the realization of any contingent spoils. This creates a tradeoff between the cost of delay and the costs of consolidating through fighting. As shown in the next section the larger the contingent spoils, the higher the cost of delay and the more likely the factions are to fight.

**IV. Comparative Statics**

Whether \(G\) consolidates power and monopolizes violence peacefully or through fighting depends on the size of the contingent spoils. The cost of fighting and the initial distribution of power also affect the likelihood of fighting. We examine each in turn.

Contingent spoils create the key tradeoff that leads to fighting in the model. Consolidating power peacefully avoids the losses due to fighting, but it takes time for \(G\) to gradually weaken and then eliminate \(R\). More specifically, \(G\) must transfer \(\Pi_R(s_k)\) to \(R\), and this takes \(q(s_k)\) periods. This delays the date at which the contingent spoils begin to flow. If the spoils are small, the costs of delay are small and \(G\) consolidates power peacefully. If the spoils are large, the cost of delay is large and \(G\) tries to consolidate power by defeating \(R\).

**Proposition 4:** There exist thresholds \(0 < \gamma < \tau\) such that \(G\) eliminates \(R\) as quickly

\[21\] If \(B(s_k) = V - \beta^n(V - f_R)\) for an integer \(n \geq 1\), then \(R\) is indifferent between accepting and rejecting offers of 1 if \(G\) tries to buy \(R\) off as quickly as possible. We disregard this case as nongeneric.
as is peacefully possible when \(0 < \gamma < \gamma^*\) and fights at either \(s_k\) or \(\tilde{s}_{k+1}\) when \(\gamma > \gamma^*\).\(^{22}\)

The higher \(G\)'s payoff during periods of fighting, \(f_G\), the lower \(G\)'s cost to fighting and the more likely the factions are to fight. More precisely, \(G\)'s payoff to fighting at \(s_k\) or at \(\tilde{s}_{k+1}\) are increasing in \(f_G\). By contrast, the cost of buying \(R\) off, \(\Pi_R(s_k)\), is independent of \(G\)'s cost of fighting. As a result, fighting becomes more likely as \(f_G\) increases and the cost of fighting declines, i.e., \(\max\{F_G(s_k), \beta F_G(\tilde{s}_{k+1})\} - \Pi_G(s_k)\) is strictly increasing in \(f_G\).

In assessing the other comparative statics, it is useful to observe that

\[
F_G(s_k) - \Pi_G(s_k) = F_G(s_k) + \Pi_R(s_k) - (1 + \beta^{q(s_k)} \gamma)V
\]

Recalling that \(\tilde{s}_{k+1} = (1, \tilde{p}_{k+1})\) and using \(1 + \beta F_R(\tilde{s}_{k+1}) = \Pi_R(s_k)\) and \(F_G(\tilde{s}_{k+1}) + F_R(\tilde{s}_{k+1}) = f_G + f_R + (1 + \gamma)V\), we can write

\[
\beta F_G(\tilde{s}_{k+1}) - \Pi_G(s_k) = \beta F_G(\tilde{s}_{k+1}) + \Pi_R(s_k) - [(1 + \beta^{q(s_k)} \gamma)V] \\
= 1 + \beta [F_G(\tilde{s}_{k+1}) + F_R(\tilde{s}_{k+1})] - (1 + \beta^{q(s_k)} \gamma)V \\
= (\beta^2 - \beta^{q(s_k)}) \gamma V - (1 - f_G - f_R).
\]

The second term in Eq (3) is the total cost of fighting at \(\tilde{s}_{k+1}\) where fighting is completely decisive and a fight to the finish only lasts one period. The first term is the difference between the payoff to the flow of contingent spoils if it starts if it starts in two periods, which it does if \(G\) buys \(R\) off at \(s_k\) and then fights at \(\tilde{s}_{k+1}\), and to the flow if it starts after \(q(s_k)\) rounds, which it does if \(G\) eliminates \(R\) peacefully.

Changes in \(f_R\) can be related to changes in the rebels’ opportunity cost of fighting. The higher the payoff during periods of fighting, the lower the opportunity cost. Lower costs (higher \(f_R\)) make fighting more likely. The larger \(f_R\), the more costly it is for the government to buy the opposition off and consolidate power peacefully. The government

\(^{22}\) The gap between \(\gamma\) and \(\gamma^*\) results from a discontinuity in \(G\)'s payoff to eliminating \(R\) as fast as is peacefully possible. If \(F_R(s_k)\) is slightly less than \((1 - \beta^m)V + \beta^m f_R\), then a small increase in \(\gamma\) may require \(G\) to take an additional period to buy \(R\) off. This postpones the contingent gain for a period and results in a discontinuous loss for \(G\) of \(\beta^m \gamma\).
must transfer more to the opposition ($\partial \Pi_R / \partial f_R > 0$), and it may take longer to do this ($q(s_k)$ is weakly increasing in $\Pi_R$). It follows from Eq (2) that $F_G(s_k) - \Pi_G(s_k)$ is increasing in $f_R$. Eq (3) implies that $\beta F_G(\bar{s}_{k+1}) - \Pi_G(s_k)$ is also increasing. An increase in $f_R$ thus makes fighting more likely.

Perhaps surprisingly, the factions are more likely to fight the stronger $R$’s initial position (the higher $p_k$). This reflects the fact that fighting depends on the difference between $G$’s payoffs to buying $R$ off and to fighting. Both of these payoffs decrease as $p_k$ increases, but the difference $\max\{F_G(s_k), \beta F_G(\bar{s}_{k+1})\} - \Pi_G(s_k)$ increases. In other words, the government is more likely to try to buy off weaker groups. Summarizing,

**Proposition 5 (Comparative Statics):** If the contingent spoils are sufficiently large, then fighting becomes more likely as the flow payoffs during periods of fighting ($f_G$ or $f_R$) increase or the opposition becomes stronger ($p_k$ increases).\(^\text{23}\)

V. Extensions

This section briefly elaborates two extensions of the model. The first highlights the importance of commitment by showing that weaker institutions which are less able to make credible commitments to future transfers make for more fighting. The second demonstrates that the less able the government is to consolidate its position in the absence of fighting, perhaps because of corruption or its limited capacity to provide public goods, the more likely the factions are to fight.

The fundamental source of inefficiency and the cause of fighting in the model is the government’s inability to commit to future divisions of the spoils coupled with its inability to commit to not exploiting a weaker opposition. Regardless of what the factions agree to today, those arrangements are always subject to renegotiation tomorrow in light of the *de facto* distribution of power that exists tomorrow. Whatever political arrangements or institutions the factions put in place today to determine the distribution of future benefits are completely ineffectual. The distribution of *de jure* power defined by those institutions has no effect on the distribution of future benefits; the distribution of benefits

\[^{23}\text{If } \beta \gamma V < 1 - f_G - f_R, \text{ then } F_G(s_k) - \Pi_G(s_k) < 0, \beta F_G(\bar{s}_{k+1}) - \Pi_G(s_k) < 0, \text{ and there is no fighting at any } s_k.\]
at a future time \( t \) is determined solely by the distribution of \textit{de facto} power \((d_t, p_t)\).\textsuperscript{24} Institutional capacity is zero in the sense that institutions lack any ability to bind or commit the factions in the future.

Suppose instead that the state has some institutional capacity which enables the government to “commit” to transferring some of the future benefits to \( R \) at least in expectation (Acemoglu and Robinson 2006, North and Weingast 1989). More specifically, suppose that \( G \) can offer to share power with \( R \) in return for \( R \)'s disarming where sharing power is an instrumental way of sharing future benefits. Implicit in this proposal (and unmodelled) are the political and institutional arrangements designed to implement this division of benefits, e.g., creating or dividing up ministries, granting regional autonomy, using quotas to allocate parliamentary seats, etc. The greater the institutional capacity of the state, the more likely these arrangements are to hold and the more likely \( R \) is to see the promised share of future benefits.\textsuperscript{25}

Formally, if \( G \) names \( E \) as the next stage, then the proposal takes the form \((z, y, E)\). As before, \( z \in [0, 1] \) is the share of the current pie on offer to which \( G \) can commit to giving \( R \). The second component \( y \in [0, (1 + \gamma)V] \) is the share of the future spoils that \( G \) promises to \( R \). If \( R \) agrees to this proposal, \( R \) obtains \( z + \beta y \) if the power-sharing arrangements hold and \( z + \beta f_R \) if the arrangements break down where, recall, \( f_R \) is \( R \)'s payoff at \( E \). The power-sharing agreement holds with probability \( \kappa_1 \). Consequently, \( R \) and \( G \)'s expected payoffs to agreeing on \((z, y, E)\) are \( \kappa_1(z + \beta y) + (1 - \kappa_1)(z + \beta f_R) = z + \beta f_R + \beta \kappa_1(y - f_R) \)

and \( 1 - z + \beta (1 + \gamma)V - \beta f_R - \beta \kappa_1(y - f_R) \).

The parameter \( \kappa_1 \) is a highly reduced-form measure of institutional capacity or commitment power. The higher \( \kappa_1 \), the more likely today’s agreements about future divisions are to hold. The analysis above focused on the case in which today’s agreements have no effect on tomorrow’s outcomes \((\kappa_1 = 0)\). By contrast, power-sharing agreements are sure

\textsuperscript{24} See Acemoglu and Robinson (2006) for a discussion of \textit{de facto} and \textit{de jure} power.

to hold when $\kappa_1 = 1$. In keeping with the idea of a weak state, assume $\kappa_1$ is small.

The effects of limited institutional capacity on equilibrium play are straightforward. Since $G$ can transfer more to $R$ in return for moving to $E$, $G$ can get $R$ to agree to $E$ when it is stronger. This shortens the time it takes to monopolize violence peacefully. Stronger institutions (higher $\kappa_1$) therefore make fighting less likely.

The second extension centers on $G$’s ability to consolidate power in the absence of fighting. The model assumes that if $G$ undertakes efforts to consolidate power by moving from $s_k$ to any $s_{k+1}$, then these efforts are sure to succeed if there is no fighting. Suppose instead that $G$’s ability to consolidate its position is uncertain even in the absence of fighting. For example the Karzai government in Afghanistan has had great difficulty consolidating its position even in areas with little or no fighting. Formally, assume that if $G$ proposes $(z_k, s_{k+1})$ at $s_k$ and $R$ accepts, play moves to stage $s_{k+1}$ with probability $\kappa_2$ and remains at $s_k$ with probability $1 - \kappa_2$. The parameter $\kappa_2$ reflects a different dimension of state capacity. The lower $\kappa_2$, the less likely the government’s efforts to consolidate are to succeed.

The lower this capacity, the more likely the factions are to fight. As long as $G$ is more likely to consolidate power in the absence of fighting, i.e., as long as $\kappa_2 > \varepsilon(1 - d_k)$, $G$ will still try to consolidate power by offering $z_k = 1$ in return for weakening $R$ as much as possible. But the less likely $G$’s efforts to consolidate are to succeed (the lower $\kappa_2$), the longer it will take to eliminate $R$ peacefully. This delays the contingent spoils and makes fighting more likely.

More formally, $R$’s payoff to accepting $(z_k, s_{k+1})$ is $z_k + \beta[\kappa_2V_R(s_{k+1}) + (1 - \kappa_2)V_R(s_k)]$. As a result, $R$’s peaceful participation constraint becomes

$$z_k + \beta[\kappa_2V_R(s_{k+1}) + (1 - \kappa_2)V_R(s_k)] 
\geq f_R + \beta d_k p_k(1 + \gamma)V + \beta(1 - d_k)[(1 - \varepsilon)V_R(s_k) + \varepsilon V_R(s_{k+1})]. \quad (PPC')$$

The key force driving consolidation in the baseline model with $\kappa_2 = 1$ is that $G$ can lower $R$’s payoff and thereby increase its own by using $z_k$ to weaken $R$. The same force
is at work when G’s ability to consolidate is uncertain as long as G is more likely to consolidate when there is no fighting. That is, G’s payoff to accepting \((z_k, s_{k+1})\) is equal to its payoff to fighting when PPC’ binds. This payoff is decreasing in \(V_R(s_{k+1})\), and \(z_k\) and \(V_R(s_{k+1})\) are inversely related as long as \(\kappa_2 > \varepsilon(1 - d_k)\). Thus, minimizing \(V_R(s_{k+1})\) means offering \(z_k = 1\).

In order to induce \(R\) to move to \(E\) peacefully, G must transfer \(\Pi_R(s_k, \kappa_2)\) to \(R\) where \(\Pi_R(s_k, \kappa_2)\) is decreasing in \(\kappa_2\). Because G must transfer more when institutions are weak (\(\kappa_2\) is small), it takes longer to eliminate \(R\) peacefully. Arguing as above, it is easy to show that \(F_G(s_k, \kappa_2) - \Pi_G(s_k, \kappa_2)\) and \(\beta F_G(\bar{s}_{k+1}, \kappa_2) - \Pi_G(s_k, \kappa_2)\) are decreasing in \(\kappa_2\). Hence, the lower the probability of consolidating \(\kappa_2\), the more likely the factions are to fight.

VI. Related Work

The present model is closely related to other models of civil war and of coercion. As suggested above, the key difference between the model analyzed here and other models of civil war is that the shift in power is endogenous. Suppose, for example, that the game described above only lasted two periods \((t = 0, 1)\); fighting was sure to be decisive \((d_0 = d_1 = 1)\); there were no contingent spoils \((\gamma = 0)\); and the distribution of power exogenously shifted against the opposition \((p_0 > p_1)\). Since the shift in power is exogenous, G’s proposals are limited to specifying how the current pie will be divided, and \(R\) decides whether to accept or fight. This specification corresponds to what Fearon (1998) describes as his “toy” model of the commitment problem underlying ethnic conflict. Fighting erupts in this setting when the adverse shift in the rebels’ power is so large that the government cannot offer the rebels enough to compensate them for agreeing to

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26 Rewrite the binding PPC’ as \(z_k + \beta[\kappa_2 - \varepsilon(1 - d_k)]V_R(s_{k+1}) = f_R + \beta d_k p_k (1 + \gamma)V + \beta[(1 - d_k)(1 - \varepsilon) - (1 - \kappa_2)]V_R(s_k)\). The right side of this equality is independent of \(G\)’s proposal. Hence, the larger \(z_k\), the smaller \(V_R(s_{k+1})\) as long as \(\kappa_2 > \varepsilon(1 - d_k)\).

27 As in the baseline game, \(\Pi_R\) is the continuation value \(V_R(s_k)\) obtained from solving \(V_R(s_k) = 1 + \beta V_R(s_{k+1})\) and the binding peaceful participation constraint for \(V_R(s_k)\). This gives \(\Pi_R(s_k, \kappa_2) = \kappa_2[f_R + \beta d_k p_k (1 + \gamma)V - \varepsilon(1 - d_k)]/[\kappa_2(1 - \beta(1 - d_k)(1 - \varepsilon))] - \varepsilon(1 - d_k)[1 - \beta(1 - \kappa_2)]\) where \(\partial \Pi_R(s_k, \kappa_2)/\partial \kappa_2 < 0\).
be much weaker in the next period.

Fearon (2004) develops a more general infinite-horizon stochastic game in his discussion of why some civil wars last so long. At the start of peace-periods, the government makes a take-it-or-leave-it offer to the rebels. If the government is weak, the rebels can accept or reject by fighting.²⁸ If the rebels accept, then (with high probability) the government overcomes its temporary weakness, i.e., there is an exogenous shift in the distribution of power against the rebels. If by contrast the rebels fight, either the government or the rebels win or fighting is indecisive and play moves to the next stage. If neither faction prevails, the distribution of power remains unchanged and the government continues to be weak. Fighting, in other words, is sure to prevent an adverse shift in the distribution of power. As in the toy model, fighting occurs when the government is unable to offer the rebels enough to compensate them for agreeing to the exogenous adverse shift.

Most directly, Powell (2012) analyzes a game parallel to the one studied here except that the pattern of state consolidation and shifting power is exogenous. That is, there are \(N+1\) exogenously specified stages \(\{s_n\}_{n=0}^N\) where \(s_n = (d_n, p_n)\). In any stage \(s_n\) for \(n < N\), \(G\) makes a take-it-or-leave-it proposal \(z_n \in [0, 1]\). If \(R\) accepts, play moves to stage \(s_{n+1}\). If \(R\) fights, play moves to \(s_{n+1}\) with probability \(\varepsilon (1 - d_n)\). (Once play reaches \(s_N\) the state is fully consolidated and distribution of power remains at \(s_N\) in all subsequent periods.) Powell links the pattern of shifting power to the pattern of equilibrium fighting, showing that fighting occurs in any \(s_n\) whenever the shift in power from \(s_n\) to \(s_{n+1}\) is sufficiently large.

Crost, Felter, and Johnson (2011) develop a model in which a successful development project leads to a large adverse shift in power against an insurgent group. Using a regression-discontinuity design and data from a large development program in the Philippines, they find that municipalities eligible for the program suffered a significant increase

²⁸ To simplify matters, Fearon assumes that the rebels must accept the government’s offer when the government is strong. A more complicated approach is to allow the rebels the option of fighting but then parameterize the game game so that this option is strictly dominated when the government is strong (see, for example, the models of political transition in Acemoglu and Robinson (2002, 2006).)
in violence which only lasted for the duration of the project.

The central issue in all of these analyses, as well as in the present one, is that large shifts in the distribution of power create commitment problems that lead to fighting.\textsuperscript{29} When the shifts are exogenous, fighting results. When the shifts are endogenous, the government can avoid a fight if it chooses.

The role that coercive power plays in the present analysis is analogous to the role that it plays in Acemoglu and Wolitzky’s (2011) model of labor coercion. The key to their results is that the principal, who is a producer, can affect the agent’s (the laborer’s) participation constraint by investing in guns. In particular, spending more on guns relaxes the agent’s participation constraint by lowering the agent’s payoff to rejecting the principal’s offer (by, for example, running away).\textsuperscript{30} However, buying guns is costly, and this limits the principal’s willingness to use guns to lower the agent’s reservation value.

Here, the (unmodelled) steps $G$ can take to consolidate power are assumed to be costless in order to simplify the analysis. Rather, $G$’s ability to induce $R$ to accept a proposal by lowering its reservation value is limited by $G$’s inability to offer more than $z_k = 1$.

VII. Some Empirics

Three main forces drive the dynamics of consolidation in the model. Coercive power gives the government an incentive to consolidate power and monopolize violence. Commitment problems created by large shifts in the distribution of power limit the rate at which peaceful consolidation can occur and thus delay the realization of any contingent spoils. The larger these spoils, the higher the cost of delay and the more likely the government is to try to consolidate through fighting.

\textsuperscript{29} More generally, the commitment problem and resulting inefficiency arise from large changes in the actors’ continuation payoffs. These changes are due to shifts in the distribution of power in the models above. But they might be driven by temporary shocks to the relative cost of fighting as in Acemoglu and Robinson’s (2001, 2002, 2006) models of political transitions or Chassang and Padro’s (2009) model of civil war. See Powell (2004) for a discussion of this mechanism.

\textsuperscript{30} The contrasts with Chwe’s (1990) model of slavery where the agent’s reservation value is exogenous.
This section discusses some cases and more general empirical findings that illustrate these forces at work or suggest that they are. In Iraq, the government has gradually weakened and demobilized the Sunni Awakening militias. The contingent spoils in this case also appear to be small. By contrast, the discovery of oil in Sudan created large contingent spoils and was a major factor leading to the second civil war (1983-2005). More generally, contingent spoils provide a natural explanation for Lujala’s (2010) and Ross’ (2012) finding that onshore oil production is associated with civil war onset but offshore production is not.

Before discussing the cases, it is useful to explain the focus on oil. Contingent spoils are the returns from any increased economic activity resulting from the higher level of security and protection that comes with the monopolization of violence. To the extent that these returns come from higher natural resource rents, higher contingent rents will be associated with more fighting. The contingent aspect of these returns is perhaps most evident in the case of oil the exploitation of which requires a large investment in often highly vulnerable infrastructure. Moreover, there is substantial case-study and statistical work on the relationship between oil and civil war onset (e.g., Fearon and Laitin 2003; Collier and Hoefller 2004; Ross 2004, 2012; Fearon 2005; Humphreys 2005; Le Billon 2005; Dube and Vargas 2011).

In September 2006, several tribes in the Iraqi province of Al-Anbar turned on Al-Qaida and allied with the United States in what became known as the “Sunni Awakening.” The “Sons of Iraq” movement quickly spread across Iraqi in areas dominated by Arab Sunnis. Responding to a call from their tribal leaders, Sunni’s joined the Iraqi army and police in unprecedented numbers. Sunni tribes also established the Sahwa (Awakening) militias to fight Al-Qaida. The United States provided these militias with substantial military assistance, protection, and financial support (Kilcullen 2007, West 2008, Wilbanks and Karsh 2010, Cigar 2011, Benraad 2012). Violence dropped dramatically over the next year. United States military fatalities fell from a monthly high of 127 in May 2007 to 23 in December of that year. Civilian fatalities declined over the same period from 1700 per

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31 The kurds in the north are also Sunni but constituted a very different group.
month to 500 (Biddle, Friedman, and Shapiro 2012).

Iraqi leaders were wary of the Sahwa militias from the start (Wilbanks and Karsh 2010, Hopkins 2012). As American forces began to withdraw from Iraq and the government assumed responsibility for dealing with the Sahwa leaders, Al-Maliki expressed the concern “that the government needed to ensure that it had a monopoly over armed force and announced that it would limit Sahwa powers of arrest” (Cigar 2011, 65; Benraad 2012).\textsuperscript{32} There is also some indication that Al-Maliki would have liked to do this very rapidly if he could. In January 2009, he is reported to have said that he “wanted to close out the [Sahwa] file in just three months” (Saeed 2009; Cigar 2011, 70).

Despite this preference, events unfolded more slowly. In the ensuing months, the Al-Maliki government took a number of steps which gradually weakened the Sunni militias. Sunni leaders were arrested. New organizations under the direct control of the central government were set up to circumvent and undermine the authority of tribal leaders. Pay and benefits promised to the Sahwa fighters were often delayed and cut. The government stopped paying for the tribal leader’s bodyguards in 2010 which increased the leaders’ vulnerability to assassination attempts. The government also tried to disarm some local militias. (Al Jazeera, 2010, Cigar 2011, Benraad 2012). All of this led to a gradual depletion of Sahwa forces. In Diyala, for example, the Sahwa ranks fell from a peak of 14,000 to about 6,000 in 2010. Integration into Iraqi security forces and other government jobs, which the government had promised to do, accounted for only a quarter of this decline (Cigar 2011, 77).

Importantly, the Iraqi government continued to provide some resources to Sunni leaders throughout this process and to integrate some Sahwa fighters however slowly. For their part, these leaders frequently warned that if the government failed to live up to its earlier commitments (usually about the government’s 2008 promise to incorporate 20 percent of the Sahwa fighters into the security forces and find the others government jobs), these fighters would turn on the government and rejoin Al-Qaida (Nordland and Rubin 2009;\textsuperscript{32} As specified in the Status of Forces Agreement between Iraq and the United States, which the Iraqi Parliament approved in November 2008, the United States withdrew its troops from Iraqi cities by June 30, 2009 and from Iraq by the end of 2011.)
As for the contingent spoils, Iraq derives most of its income from oil. Oil revenues funded about 95 percent of Iraq’s 2012 budget (Kami 2012, Katzman 2012). But there are virtually no proven reserves in the Sunni-dominated areas where the Sawha militias stood up (see Map 1). The militias posed little obstacle to the development of Iraq’s oil fields, and, as a result, the opportunity cost of gradually weakening and demobilizing these militias was likely to be low.

[Map 1]

[The case of Sudan’s second civil war]

Contingent spoils also provide a natural explanation for some more systematic findings which existing theories cannot explain very well. Researchers generally find that oil is positively related to civil war onset (e.g., Fearon and Laitin 2003; Collier and Hoefler 2004, 2005; Ross 2004, 2012; Fearon 2005; Humphreys 2005; Le Billon 2005). However, Lujala (2010) and Ross (2012) show that there are significant location effects. Onshore oil makes civil war more likely, but offshore oil does not.

This is in keeping with expectations based on contingent spoils. Onshore oil and related facilities are much more vulnerable to rebel attack than offshore facilities are. Thus the latent threat posed by renewed fighting between the government and an armed opposition is larger for onshore oil.

By contrast, the two main explanations linking oil to civil war onset cannot account for these location effects (Lujala 2010, Ross 2012). In “state prize” accounts, higher resource rents make controlling the state more valuable and this leads to more fighting (Bates 2008, Besley and Persson 2010, Blattman and Miguel 2010, Bazzi and Blattman 2012). Fearon and Laitin emphasize weak institutions rather than the value of the state in explaining the relationship between oil and civil war (although they also say that “oil revenues raise the value of the ‘prize’ of controlling state power” (Fearon and Laitin 2003,
81)). That is, large oil revenues lead to weaker states, and state weakness, whether due to oil, terrain or poverty, makes civil war more likely. Whether a country’s oil wells are onshore or off is irrelevant to both of these explanations. It is the revenue, not the source, that makes controlling the state a more valuable prize or leads to weakness.

Lujala and Ross trace these location effects to the rebels’ ability to finance their operations through looting. Onshore oil is more vulnerable to rebel efforts to steal oil directly by tapping into pipelines and hijacking trucks or to raise revenues through extortion and kidnapping. This, however, is at most a partial explanation. A lower opportunity cost makes it easier to fund a rebel group. But it does not explain why the government and rebel group would subsequently engage in inefficient fighting. Why does the government not buy the rebels off? In terms of the model, larger returns to looting can be interpreted as an increase in the rebels’ payoff to fighting $f_R$. Conditional on the contingent spoils being sufficiently large, an increase in $f_R$ makes fighting more likely. But if there are no contingent spoils or they are too small, there is no fighting.

Conclusion

When the government in a weak state faces an armed opposing faction, the government has to decide whether to live with an armed opposition or to try to consolidate its power and monopolize violence by disarming it. If the latter, the government can try to disarm the opposition peacefully by buying it off or by defeating it militarily. When and why do governments choose to consolidate power and monopolize violence? How fast do they try to consolidate power? When does this lead to costly fighting rather than to efforts to eliminate the opposition by buying it off?

Three main forces drive the dynamics of consolidation in the model and provide answers to these questions. Coercive power creates an incentive for the government to consolidate power and monopolize violence. Commitment problems arising resulting from large shifts

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34 Opportunity-cost arguments are typically modeled with contest functions in which there is no explicit decision to fight and arming is treated as being equivalent to fighting (e.g., Besley and Persson 2010, Dal Bó and Dal Bó 2011). See Blattman and Miguel (2010) and Fearon (2007) on this point.
in the distribution of power limit the rate at which peaceful consolidation can occur and thus delay the realization of any contingent spoils. The larger these spoils, the higher the opportunity cost of delay and the more likely the government is to try to consolidate through fighting.

Lower opportunity costs to fighting (higher flow payoffs $f_G$ and $f_R$) make fighting more likely. The factions are also more likely to fight when the opposition is stronger ($p_k$ is higher). Greater institutional capacity to commit to future transfers or to consolidate power in the absence of fighting makes fighting less likely.

The present model endogenizes the distribution of power in a very reduced-form and asymmetric way. The government alone can shift the distribution of power. An important area for future work is a more symmetric and microfounded treatment of each actor’s efforts to shift the distribution of power in its favor.
Appendix

[Note to reader: I am midway through a very substantial revision of the paper. The body of the paper has been revised but not the appendix. The appendix does characterize the equilibrium. But there is currently a mismatch between the way that the results are described in body of the paper and in the appendix.]

The appendix proves Propositions 1 and 2. Proposition 3 is an immediate extension of Proposition 1. Let $E = \{\zeta, \sigma, \alpha\}$ be any pure-strategy MPE and take $V_i(s_k)$ for $i \in \{G, R\}$ to be $j$’s continuation payoff starting from any stage $s_k$ if the factions play according to $E$. The sequence $(z_k, s_{k+1}), (z_{k+1}, s_{k+2}), \ldots$ denotes the path of play starting from $s_k$.

Although the details of the proof of the Proposition 1 are cumbersome, the underlying intuition is straightforward. Lemma 1A establishes useful upper bounds on the factions’ continuation values if the continuation game entails fighting. Lemma 2A characterizes the equilibrium starting from any stage at which fighting is sure to be decisive, i.e., any $s_k$ with $d_k = 1$. Because fighting is decisive, $G$ lacks coercive power and the continuation payoffs and path are relatively easy to specify. The lemma also shows that $V_R(s_k)$ for $s_k = (1, p)$ is continuous in $p$. Lemma 3A shows that if the factions do not fight at $s_k$ in $E$, then $G$ can move play to a stage where fighting is sure to be decisive. More specifically, if $R$ accepts $(z_k, s_{k+1})$ at $s_k$ in $E$, then $G$ can induce $R$ to move to an $s'_{k+1} = (1, p'_{k+1})$. Lemmas 4A and 5A demonstrate that if $R$ accepts $(z_k, s_{k+1})$, then $R$’s continuation payoff at $s_{k+1}$ is equal to its continuation payoff at an $s'$ where fighting is sure to be decisive, i.e., $V_R(s_{k+1}) = V_R(s')$ where $s' = (1, p')$ for a $p' \in [0, 1)$. Since $p' < 0$ and $V_R(1, p)$ is continuous in $p$, Lemma 5A ensures that the PPC must bind at $s_k$ if $R$ accepts $(z_k, s_{k+1})$ in $E$. Proposition 1 uses this and Lemma 2A’s characterization of the continuation games starting from stages where fighting is decisive to specify equilibrium behavior and payoffs starting from any stage.

**Lemma 1A:** Suppose the factions fight in the continuation game starting from $s_k$. If $1 - f_G - f_R \leq \beta \gamma V$, then $V_G(s_k) + V_R(s_k) \leq f_G + f_R + \beta(1 + \gamma) V$. If $1 - f_G - f_R < \beta \gamma V$, then $V_G(s_k) + V_R(s_k) \leq V$.

**Proof:** If the factions fight at $s_{k+m}$, then $V_G(s_k) + V_R(s_k) = (1 - \beta^m)V + \beta^m[F_R(s_{k+m}) +
$F_G(s_{k+m})$ where $F_G(s_{k+m}) + F_R(s_{k+m}) = [f_G + f_R + \beta(1+\gamma)V[d_{k+m}(1-\varepsilon)]/[1 - \beta(1-d_{k+m})(1-\varepsilon)]$. The expression on the right is increasing in $d_{k+m}$, so $V_G(s_k) + V_R(s_k) \leq (1 - \beta^m)V + \beta^m[f_G + f_R + \beta(1+\gamma)V] = V - \beta^m(1 - f_R - f_G - \beta \gamma V)$. If $1 - f_R - f_G - \beta \gamma V \leq 0$, then $V_G(s_k) + V_R(s_k) \leq f_G + f_R + \beta(1+\gamma)V$. If $1 - f_R - f_G - \beta \gamma V > 0$, $V_G(s_k) + V_R(s_k) \leq V$. □

Lemma 2A characterizes the equilibrium starting from $s_k$ with $d_k = 1$. Note that $B(s_k) = F_R(s_k) = \Pi_R(s_k) = F_R(s_k) = f_R + \beta p_k(1 + \gamma)V$ when $d_k = 1$.

**Lemma 2A:** Let $s_k = (1, p_k)$ and assume $\gamma > 0$. Then (i) $V_R(s_k) = f_R + \beta p_k(1 + \gamma)V$; and (ii) if $p' > p$, $s = (1, p)$, $s' = (1, p')$, and $F_G(s) > \Pi_G(s)$, then $F_G(s') > \Pi_G(s')$. Assume $B(s_k) \neq V - \beta^n(V - f_R)$ for any integer $n \geq 1$. Then (iii) the factions fight at $s_k$ with $V_G(s_k) = F_G(s_k)$ if $F_G(s_k) > \Pi_G(s_k)$, and (iv) $G$ eliminates $R$ as fast as is peacefully possible if $\Pi_G(s_k) > F_G(s_k)$.

**Proof:** To establish (ii), suppose $\Pi_R(s) < V$ and $F_G(s) > \Pi_G(s)$. Since $\Pi_G(s) = (1 + \beta q(s))V - F_R(s)$, we have $F_G(s) + F_R(s) > (1 + \beta q(s))V$. Observe further that $F_G(s) + F_R(s) = F_G(s') + F_R(s') = f_G + f_R + \beta(1 + \gamma)V$. Hence, $F_G(s') > (1 + \beta q(s'))V - F_R(s') \geq (1 + \beta q(s'))V - f_R(s') = \Pi_G(s')$ where the weak inequality holds since $q(s') \geq q(s)$. Suppose alternatively that $\Pi_R(s) \geq V$. Trivially, $\Pi_R(s) \geq V$ implies $\Pi_R(s') \geq V$ and, consequently, that $\Pi_G(s') \leq 0$. This leaves leaves $F_G(s') > f_R > 0 \geq \Pi_G(s')$.

To demonstrate (i), assume first that $B(s_k) \geq V$. It follows that the factions must fight in the continuation game. If the inequality is strict, $G$ cannot transfer enough to $R$ to buy it off. If the inequality is weak, $G$ can buy $R$ by offering $z_{k+j} = 1$ for all $j \geq 0$. But this leaves $V_G(s_k) = 0 < F_R(s_k)$. This means that fighting at $s_k$ would be a profitable deviation, and this contradiction ensures that factions fight in the continuation game.

Rewriting $B(s_k) \geq V$ as $F_R(s_k) \geq V$ implies $F_G(s_k) + F_R(s_k) > V$. This is equivalent to $1 - f_R - f_G - \beta(1 + \gamma) < 0$. Lemma 1A now ensure that the factions’ payoffs to fighting in continuation game are bounded by $V_G(s_k) + V_R(s_k) \leq f_G + f_R + \beta(1 + \gamma)V$. But $V_G(s_k) \geq f_G + \beta(1 - p_k)(1 + \gamma)V$ since $G$ can always induce a fight at $s_k$. This yields $f_R + \beta p_k(1 + \gamma)V \leq V_R(s_k) \leq f_G + f_R + \beta(1 + \gamma)V - V_G(s_k) \leq f_R + \beta p_k(1 + \gamma)V$ and establishes the claim.

Now assume $B(s_k) < V$. Since $B(s_k) \neq V - \beta^n(V - f_R)$ for any integer $n \geq 1$, $G$ can obtain a payoff less than but arbitrarily close to $\Pi_G(s_k)$ eliminating $R$ as quickly as
peacefully possible. Showing this will establish that $V_G(s_k) \geq \Pi_G(s_k)$.

To establish that $G$ can obtain a payoff arbitrarily less than $\Pi_G(s_k)$, let $s'_{k+1} = (1, p'_{k+1} + \delta_1)$ where $\delta_1 > 0$ and $p'_{k+1}$ satisfies $1 + \beta[f_R + \beta p'_{k+1}(1 + \gamma) V] = f_R + \beta p_k(1 + \gamma) V$. Then $R$ is sure to accept $s'_{k+1}$ at $s_k$ since $1 + \beta V_R(s'_{k+1}) \geq 1 + \beta[f_R + \beta (p'_{k+1} + \delta_1)(1 + \gamma) V] > f_R + \beta p_k(1 + \gamma) V$. Repeating the argument, $R$ is sure to accept $(1, s'_{k+2})$ at $s_{k+1}$ where $s'_{k+2} = (1, p'_{k+2} + \delta_2)$, $\delta_2 > 0$, and $1 + \beta[f_R + \beta p'_{k+2}(1 + \gamma) V] = f_R + \beta (p'_{k+1} + \delta_1)(1 + \gamma) V$. By choosing $\delta_j$ small enough we can continue in this way until $f_R + \beta (p'_{k+q(s_k) - 1} + \delta_{q(s_k) - 1})(1 + \gamma) V < 1 + \beta f_R$ at which point $R$ is sure to accept $(f_R + \beta (p'_{k+q(s_k) - 1} + \delta_{q(s_k) - 1})(1 + \gamma) V + \delta_{q(s_k)}, E)$ for any $\delta_{q(s_k)} > 0$.

Let $V'_G(s_k, \overline{\delta})$ and $V'_R(s_k, \overline{\delta})$ denote the factions’ continuation payoffs with $\overline{\delta} \equiv \max\{\delta_1, ..., \delta_{q(s_k)}\}$. Then $V'_R(s_k, \overline{\delta}) > V'_R(s_k, 0) = f_R + \beta p_k(1 + \gamma) V$ and continuity ensures $\lim_{\overline{\delta} \to 0} V'_R(s_k, \overline{\delta}) = V'_R(s_k, 0)$. Since $R$ agrees to move to $E$ after $q(s_k)$ offers, $V'_G(s_k, \overline{\delta}) + V'_R(s_k, \overline{\delta}) = V + \beta^{q(s_k)} \gamma V$. This gives $V'_G(s_k, \overline{\delta}) = V + \beta^{q(s_k)} \gamma V - V'_R(s_k, \overline{\delta})$. Since this cannot be a profitable deviation $V_G(s_k) \geq V'_G(s_k, \overline{\delta})$ for all $\overline{\delta} > 0$. This yields the lower bound $V_G(s_k) \geq V'_G(s_k, 0) = V + \beta^{q(s_k)} \gamma V - V'_R(s_k, 0) = V + \beta^{q(s_k)} \gamma V - B(s_k) = \Pi_G(s_k)$.

Claim (i) follows immediately if the continuation game is peaceful. Since $V_G(s_k) + V_R(s_k) \leq (1 + \beta^{q(s_k)} \gamma) V$, we have $f_R + \beta p_k(1 + \gamma) V \leq V_R(s_k) \leq (1 + \beta^{q(s_k)} \gamma) V - V_G(s_k) \leq (1 + \beta^{q(s_k)} \gamma) V - \Pi_G(s_k) \leq f_R + \beta p_k(1 + \gamma) V$ where the last inequality also uses the fact $B(s_k) = \Pi_R(s_k) = f_R + \beta p_k(1 + \gamma) V$.

Suppose alternatively that the factions fight in the continuation game. If $1 - f_G - f_R - \beta(1 + \gamma) V \leq 0$, Lemma 1A gives $V_G(s_k) + V_R(s_k) \leq f_G + f_R + \beta(1 + \gamma) V$ and, as a result, $V_G(s_k) + V_R(s_k) \leq F_G(s_k) + F_R(s_k)$. This leaves $0 \leq V_R(s_k) - F_R(s_k) \leq F_G(s_k) - V_G(s_k) \leq 0$. If, by contrast, $1 - f_G - f_R - \beta(1 + \gamma) V > 0$, then $V_G(s_k) + V_R(s_k) \leq V$. Using $V_G(s_k) \geq \Pi_G(s_k) = (1 + \beta^{q(s_k)} \gamma) V - F_R(s_k)$ gives the contradiction $V_R(s_k) \leq V - V_G(s_k) \leq -\beta^{q(s_k)} \gamma V + F_R(s_k)$. This results in the contradiction $V_R(s_k) < F_R(s_k)$ and establishes claim (i).

Arguing by contradiction to establish claim (iii), assume $F_G(s_k) > \Pi_G(s_k)$ and $G$ eliminates $R$ peacefully. Since $G$’s payoff to eliminating $R$ peacefully is bounded above by its payoff to eliminating $R$ as fast as is peacefully possible, $V_G(s_k) \leq \Pi_G(s_k)$. But
This yields the contradiction. Hence the factions must fight in the continuation game.

To see the factions fight at \( s_k \), assume they fight at \( s_{k+j} \). Then \( V_G(s_k) = \sum_{i=0}^{j-1} \beta^i(1 - z_{k+i}) + \beta^j F_G(s_{k+j}) \). Repeatedly using \( V_R(s_{k+i}) = z_{k+i} + \beta V_R(s_{k+i+1}) \) in the previous equality gives \( V_G(s_k) = (1 - \beta^j)V - V_R(s_k) + \beta^j [F_G(s_{k+j}) + F_R(s_{k+j})] \leq (1 - \beta^j)V - [f_R + \beta p_k(1 + \gamma)V] + \beta^j [f_G + f_R + \beta(1 + \gamma)V] \). This implies \( V_G(s_k) - F_G(s_k) \leq (1 - \beta^j)[V - f_G - f_R - \beta(1 + \gamma)V] \).

If the factor in brackets is negative, then the factions must fight at \( j = 0 \). Otherwise fighting at \( s_k \) rather than at \( s_{k+j} \) for \( j \geq 1 \) would be a profitable deviation. To see that this factor is negative, substitute \( \Pi_R(s_k) = f_R + \beta p_k(1 + \gamma)V \) in \( F_G(s_k) > \Pi_G(s_k) \) to obtain \( f_G + \beta(1 - p_k)(1 + \gamma)V > (1 + \beta^q(s_k))\gamma V - \Pi_R(s_k) \) or \( f_G + f_R + \beta(1 + \gamma)V > V \).

As for claim (iv), observe that it holds vacuously when \( \Pi_R(s_k) \geq V \) as this implies \( \Pi_G(s_k) \leq 0 < F_G(s_k) \). Assume then that \( \Pi_R(s_k) < V \). Since \( \Pi_R(s_k) = B(s_k) \neq V - \beta^n(V - f_R) \) for any integer \( n \geq 1 \), \( G \) can obtain a payoff arbitrarily close to \( \Pi_G(s_k) \) by eliminating \( R \) as fast as is peacefully possible. Hence, \( V_G(s_k) > \Pi_G(s_k) - \delta \) for any \( \delta > 0 \). But his proof of Lemma 1 shows that \( \Pi_G(s_k) \) is a strict upper bound on \( G \)'s payoff to eliminating \( R \) peacefully but not as fast as possible. This implies that \( G \) must either eliminate \( R \) as fast as peacefully possible or fight in the continuation game.

Suppose the factions fight. The assumption \( \Pi_G(s_k) > F_G(s_k) \) and the fact that \( V_G(s_k) \) is arbitrarily close to \( \Pi_G(s_k) \) ensure \( V_G(s_k) > F_G(s_k) \). Assume \( 1 - f_G - f_R - \beta \gamma V \leq 0 \). Then \( V_G(s_k) + V_R(s_k) \leq F_G(s_k) + F_R(s_k) \) by Lemma 1A. But \( V_R(s_k) = F_R(s_k) \) by claim (ii). This yields the contradiction \( V_G(s_k) \leq F_G(s_k) \).

Assume instead that \( 1 - f_G - f_R - \beta \gamma V > 0 \). This and fighting in the continuation game mean \( V_G(s_k) + V_R(s_k) \leq V \). Claim (ii) then implies \( V_G(s_k) \leq V - F_R(s_k) \). But \( 1 - f_G - f_R - \beta \gamma V > 0 \) is equivalent to \( F_G(s_k) + F_R(s_k) > V \). We now the contradiction \( V_G(s_k) \leq V - F_R(s_k) < F_G(s_k) \) which means that \( G \)'s fighting at \( s_k \) is a profitable deviation. □

Now consider stages at which \( d_k < 1 \). Lemma 3A shows that if \( R \) accepts \( (z_k, s_{k+1}) \) at \( s_k \) in an MPE, then \( G \) can move play to a stage in which fighting is decisive. That is, \( G \)
can make a possibly out of equilibrium proposal \((z'_k, s'_{k+1})\) which \(R\) is sure to accept and in which \(d'_{k+1} = 1\).

**Lemma 3A:** If \(R\) accepts \((z_k, s_{k+1})\) at \(s_k\) in \(E\) with \(d_k < 1\), then there exists a \(z'_k \in [0, 1]\) and an \(s'_{k+1} = (1, p'_{k+1})\) such that \(R\) accepts \((z'_k, s'_{k+1})\).

**Proof:** Assume \(V_R(s_{k+1}) < f_R + \beta(1 + \gamma)V\). Since \(V_R(s_{k+1}) \geq f_R\), there exists a \(\overline{\rho} \in [0, 1]\) such that \(V_R(s_{k+1}) = f_R + \beta\overline{\rho}(1 + \gamma)V\). Moreover, \(V_R(s'_{k+1}) = f_R + \beta\overline{\rho}(1 + \gamma)V\) by Lemma 2A where \(s'_{k+1} \equiv (1, \overline{\rho})\). \(R\)’s acceptance of \((z_k, s_{k+1})\) means that PPC holds. Substituting \(f_R + \beta\overline{\rho}(1 + \gamma)V\) for \(V_R(s_{k+1})\) gives \(z_k + \beta[f_R + \beta\overline{\rho}(1 + \gamma)V] = V_R(s_k) \geq f_R + \beta d_k p_k (1 + \gamma)V + \beta(1 - d_k)[(1 - \varepsilon)V_R(s_k)] + \varepsilon[f_R + \beta\overline{\rho}(1 + \gamma)V]\]. Hence, \(R\) is sure to accept \((z_k, (1, \overline{\rho} + \delta))\) for an arbitrarily small \(\delta > 0\).

Now assume \(V_R(s_{k+1}) \geq f_R + \beta(1 + \gamma)V\). If \(1 + \beta[f_R + \beta(1 + \gamma)V] > V_R(s_k)\), then \(R\) strictly prefers \((1, (1 - \delta))\) for \(\delta\) small enough since \(1 + \beta[f_R + \beta(1 + \gamma)V] > V_R(s_k) \geq f_R + \beta d_k p_k (1 + \gamma)V + \beta(1 - d_k)[(1 - \varepsilon)V_R(s_k)] + \varepsilon[f_R + \beta(1 + \gamma)V]\). The remainder of the proof establishes that \(1 + \beta[f_R + \beta(1 + \gamma)V] > V_R(s_k)\). (This bound will also be useful in the proof of Lemma 5A.)

To establish this bound on \(V_R(s_k)\), recall that Lemma 1A implies \(V_G(s_j) + V_R(s_j) \leq \max\{V, f_G + f_R + \beta(1 + \gamma)\}\) if the factions fight in the continuation game starting from \(s_j\). Since \(V_G(s_j) \geq f_G, V_R(s_j) \leq \max\{V - f_G, f_R + \beta(1 + \gamma)V\}\). If the factions do not fight in the continuation game, then \(V_R(s_j) \leq V\). Hence, \(V_R(s_j) \leq \max\{V, f_R + \beta(1 + \gamma)V\}\) regardless of the way the continuation game ends.

\(R\)’s acceptance of \(z_k\) then ensures \(V_R(s_k) = z_k + \beta V_R(s_{k+1}) \leq z_k + \beta \max\{V, f_R + \beta(1 + \gamma)V\}\). The bound on \(V_R(s_k)\) follows immediately if \(z_k = 0\). If \(f_R + \beta(1 + \gamma)V \geq V, 1 + \beta[f_R + \beta(1 + \gamma)V] > \beta \max\{V, f_R + \beta(1 + \gamma)V\} \geq V_R(s_k)\). If \(f_R + \beta(1 + \gamma)V < V, \) then \(\beta V \geq V_R(s_k)\) and we are done if \(1 + \beta[f_R + \beta(1 + \gamma)V] > \beta V\). But this inequality is equivalent to \(1 - \beta + \beta f_R + \beta^2 \gamma > 0\) which clearly holds.

Taking \(z_k > 0\), there are three cases to consider: (i) the factions fight at \(s_{k+1}\); (ii) the factions do not fight at \(s_{k+1}\) with \(V_R(s_k) + V_G(s_k) \leq V\); and (iii) the factions do not fight at \(s_{k+1}\) with \(V_R(s_k) + V_G(s_k) > V\). To establish that \(1 + \beta[f_R + \beta(1 + \gamma)V] > V_R(s_k)\) in case (i), note that an upper bound on what \(R\) can obtain if it fights at \(s_{k+1}\) is one in stage \(k\) and then the payoff to fighting in the best possible circumstances in the next
stage. This leaves $V_R(s_k) \leq 1 + \beta[f_R + \beta(1 + \gamma)V]$. Moreover, this inequality must be strict. If not, then $V_G(s_k) \leq \beta f_G$ since $V_R(s_k) + V_G(s_k) \leq 1 + \beta[f_R + f_G + \beta(1 + \gamma)V]$. But this means that $G$ could have profitably deviated by fighting at $s_k$. This contradiction establishes the claim in case (i).

To establish $1 + \beta[f_R + \beta(1 + \gamma)V] > V_R(s_k)$ in cases (ii) and (iii), it suffices to show that $V_R(s_k) \geq V_R(s_{k+1})$. To see why, note that since $z_k > 0$, PPC must bind. Consequently, $V_R(s_k) = f_R + \beta d_k p_k (1 + \gamma)V + \beta(1 - d_k)[(1 - \varepsilon)V_R(s_k) + \varepsilon V_R(s_{k+1})]$. This equation and $V_R(s_k) \geq V_R(s_{k+1})$ yield $V_R(s_k) \leq [f_R + \beta d_k p_k (1 + \gamma)V]/[1 - \beta(1 - d_k)]$. The expression on the right takes on its maximum value at $d_k = p_k = 1$. This leaves $V_R(s_k) < f_R + \beta(1 + \gamma)V$ since $d_k < 1$. $R$’s acceptance of $z_k$ also means that $V_R(s_k) \leq 1 + \beta V_R(s_{k+1})$. Using $V_R(s_k) \geq V_R(s_{k+1})$ again yields $V_R(s_k) \leq 1 + \beta V_R(s_k) < 1 + \beta[f_R + \beta(1 + \gamma)V]$. To show that $V_R(s_k) \geq V_R(s_{k+1})$ in case (ii), suppose the contrary. Then $V_R(s_k) < V_R(s_{k+1})$ and $V_R(s_k) + V_G(s_k) \leq V$ where, recall, the second inequality is one of the conditions defining the case. It follows that $G$ can profitably deviate at $s_k$ by slowing things down by repeating $s_k$ rather than moving on to $s_{k+1}$. To establish this, define $z'$ so that PPC binds if $G$ proposes $(z', s_k)$ at $s_k$. Then $z'$ satisfies $z' + \beta V_R(s_k) = f_R + \beta d_k p_k (1 + \gamma)V + \beta(1 - d_k)[(1 - \varepsilon)V_R(s_k) + \varepsilon V_R(s_k)]$.

Observe first that $z' < 1$. This follows by observing that $R$’s indifference between $(z_k, s_{k+1})$ and fighting along with the definition of $z'$ imply $z' + \beta V_R(s_k) = V_R(s_k) - \beta \varepsilon (1 - d_k)[V_R(s_{k+1}) - V_R(s_k)]$. By assumption $V_R(s_{k+1}) > V_R(s_k)$, so $z' + \beta V_R(s_k) < V_R(s_k) - \delta$ for a $\delta > 0$. Accordingly, $z' + \delta < (1 - \beta)V_R(s_k) < 1$ because $V_R(s_k) < V_R(s_k) + V_G(s_k) \leq V$.

It follows that $R$ is sure to accept $(\max\{0, z' + \delta\}, s_k)$. To see that this is a profitable deviation, note that $G$’s payoff to making this proposal is $V'_G(s_k) = 1 - \max\{0, z' + \delta\} + \beta V_G(s_k)$. If $z' + \delta > 0$, we have $V'_G(s_k) = 1 - z' - \delta + \beta V_G(s_k)$. This along with $z' + \delta < (1 - \beta)V_R(s_k)$ yield $V'_G(s_k) - V_G(s_k) > 1 - (1 - \beta)[V_R(s_k) + V_G(s_k)]$. But $V_G(s_k) + V_R(s_k) \leq V$, so $1 - (1 - \beta)[V_G(s_k) + V_R(s_k)] \geq 0$. Hence, $V'_G(s_k) - V_G(s_k) > 0$ which means that $G$ has a profitable deviation.

Suppose alternatively that $z' + \delta \leq 0$. Then $V'_G(s_k) = 1 + \beta V_G(s_k)$ or, equivalently, $V'_G(s_k) - V_G(s_k) = 1 - (1 - \beta)V_G(s_k) > 0$ where the strict inequality follows from the fact
that $V_G(s_k) + f_R \leq V_R(s_k) + V_G(s_k) \leq V$. Once again, $G$ has a profitable deviation and this contradiction ensures that $V_R(s_k) \geq V_R(s_{k+1})$ and consequently $1 + \beta[f_R + \beta p_k (1+\gamma)V] > V_R(s_k)$ in case (ii).

Arguing by contradiction to establish that $V_R(s_k) \geq V_R(s_{k+1})$ in case (iii), suppose the factions do not fight at $s_{k+1}$, $V_R(s_k) + V_G(s_k) > V$, and $V_R(s_k) < V_R(s_{k+1})$. Because $V_R(s_k) + V_G(s_k) > V$, the factions must either fight in the continuation game or $G$ must eliminate $R$ peacefully. Either way, play must reach $E$ in finitely many rounds. Assume $s_{k+m} = E$. Since $R$ accepts $(z_{k+1}, s_{k+2})$ at $s_{k+1}$, $m \geq 2$. If $R$ agrees to move to $E$, then $V_R(s_{k+m-1}) \leq 1 + \beta f_R$. If the factions fight at $s_{k+m-1}$, we can repeat the argument made in case (i) to show that $1 + \beta[f_R + \beta (1+\gamma)V] > V_R(s_{k+m-1})$. Hence, $1 + \beta[f_R + \beta (1+\gamma)V] > V_R(s_{k+m-1})$ whether or not the factions fight in the continuation game. We are done if $V_R(s_{k+m-1}) \geq V_R(s_{k+1})$ since $V_R(s_{k+1}) > V_R(s_{k+1})$ ensures $1 + \beta[f_R + \beta (1+\gamma)V] > V_R(s_{k+1})$.

Suppose therefore that $V_R(s_{k+1}) > V_R(s_{k+m-1})$. Then $R$’s continuation values $V_R(s_k), V_R(s_{k+1}), \ldots, V_R(s_{k+m-1})$ must peak at some stage. We show that $G$ can profitably deviate by speeding things up by skipping this peak stage. Formally, since $R$ accepts $(z_k, s_{k+1})$ and $(z_{k+1}, s_{k+2})$, $V_G(s_k) + V_R(s_k) > V$, and $V_R(s_k) < V_R(s_{k+1})$, there must exist an $s_n, s_{n+1}$, and $s_{n+2}$ such that $V_R(s_n) < V_R(s_{n+1})$ and $V_R(s_{n+1}) \geq V_R(s_{n+2})$ for an $n$ satisfying $k \leq n \leq k + m - 3$. To verify this, suppose no such $n$ exists. Since the claim does hold for $n = k$ and $V_R(s_k) < V_R(s_{k+1})$, it must be that $V_R(s_{n+1}) < V_R(s_{n+2})$. This and the fact that the claim does not hold for $k + 1$ implies $V_R(s_{n+2}) < V_R(s_{n+3})$. Repeating the argument yields the contradiction $V_R(s_{k+1}) < V_R(s_{k+2}) < \cdots < V_R(s_{k+m-1})$.

To see that $G$ can profitably deviate by speeding things up by skipping $s_{n+1}$, define $z' \equiv V_R(s_n) - \beta V_R(s_{n+2})$. Then $z' < 1$ since $1 \geq V_R(s_{n+1}) - \beta V_R(s_{n+2}) > V_R(s_n) - \beta V_R(s_{n+2})$ where $R$’s acceptance of $(z_{n+1}, s_{n+2})$ guarantees that the weak inequality holds. $R$’s acceptance of $(z_n, s_{n+1})$ ensures $0 \leq V_R(s_n) - \beta V_R(s_{n+1}) \leq V_R(s_n) - \beta V_R(s_{n+2})$. Hence, $z' \in [0, 1)$ and is therefore feasible.

$R$ is sure to accept $(z' + \delta, s_{n+2})$ at $s_n$ for any $\delta > 0$ since $z' + \beta V_R(s_{n+2}) = V_R(s_n) \geq f_R + \beta d_n p_n (1 + \gamma)V + \beta (1 - d_n) [(1 - \varepsilon) V_R(s_n) + \varepsilon V_R(s_{n+1})] + \varepsilon V_R(s_{n+1})$ and $V_R(s_{n+1}) \geq V_R(s_{n+2})$. Now consider $G$’s payoff to proposing $(z' + \delta, s_{n+2})$ and then follows $E$. $G$ obtains $V_G'(s_n) = \text{argmax}_{s_n} \left( V_G(s_n) + \beta \left[ \min \{ f_R + \beta p_k (1+\gamma)V, f_R + \beta (1 - d_n) [(1 - \varepsilon) V_R(s_n) + \varepsilon V_R(s_{n+1})] + \varepsilon V_R(s_{n+1}) \} \right] \right)$.
1 - z' - δ + βV_G(s_{n+2}) = 1 - V_R(s_n) + βV_R(s_{n+2}) + βV_G(s_{n+2}) - δ. This leaves V'_G(s_n) - V_G(s_n) = 1 - [V_R(s_n) + V_G(s_n)] + β[V_R(s_{n+2}) + βV_G(s_{n+2})] - δ. Since R accepts (z_j, s_{j+1}) for k ≤ j ≤ k + m - 1, V_R(s_j) + V_G(s_j) = 1 + β[V_R(s_{j+1}) + V_G(s_{j+1})] for k ≤ j ≤ k + m - 2. Hence, V'_G(s_n) - V_G(s_n) = [V_R(s_n) + V_G(s_n) - 1]/β - [V_R(s_n) + V_G(s_n)] - δ. That V_R(s_n) + V_G(s_n) > V ensures that we can choose δ > 0 small enough so that V'_G(s_n) - V_G(s_n) > 0 and z' + δ < 1. G therefore has a profitable deviation, and this contradiction ensures V_R(s_k) ≥ V_R(s_{k+1}) in case (iii). □

Lemma 3A showed that G can move play to stages where fighting is sure to be decisive. The next lemma demonstrates that if G is going to provoke a fight in the next round, it first moves play to a stage at which fighting is sure to be decisive.

**Lemma 4A:** If d_k < 1, R accepts (z_k, s_{k+1}) at s_k, and the factions fight at s_{k+1} in E, then s_{k+1} = (1, p_{k+1}) for a p_{k+1} ∈ [0, 1).

**Proof:** Arguing by contradiction to show that d_{k+1} = 1, assume that R accepts (z_k, s_{k+1}), the factions fight at s_{k+1}, and d_{k+1} < 1. Then G has a profitable deviation. To construct this deviation, observe first that V_G(s_k) + V_R(s_k) = 1 + β[f_R + f_G + βd_{k+1}(1 + γ)V + βε(1 - d_{k+1})(1 + γ)V]/[1 - β(1 - d_{k+1})(1 - ε)] < 1 + β[f_G + f_R + β(1 + γ)V] - δ for a sufficiently small δ > 0 where the strict inequality holds because d_{k+1} < 1.

F_R(s_{k+1}) is increasing in p_{k+1} and increasing in d_{k+1} at p_{k+1} = 1. This and d_{k+1} < 1 imply f_R + β(1 + γ)V > F_R(s_{k+1}) = V_R(s_{k+1}) > f_R. Thus, there exists a p' ∈ (0, 1) such that V_R(s_{k+1}) = f_R + βp'(1 + γ)V. Since R accepts (z_k, s_{k+1}), PPC holds at (z_k, s_{k+1}) and consequently at (z_k, (1, p'))). This means that R is sure to accept (z_k, (1, p' + δ')) for any δ' > 0. If G offers (z_k, (1, p' + δ')), and then fights, V'_G(s_k) + V'_R(s_k) = 1 + β[f_R + f_G + β(1 + γ)V] > V_G(s_k) + V_R(s_k) + δ. This leaves V'_G(s_k) - V_G(s_k) = V_R(s_k) - V'_R(s_k) + δ. But V'_R(s_k) = z_k + β[f_R + β(p' + δ')(1 + γ)] = V_R(s_k) + β^2δ'(1 + γ)V. Consequently, V'_G(s_k) - V_G(s_k) > δ - β^2δ'(1 + γ)V. Taking δ and δ' sufficiently small with δ > β^2δ'(1 + γ)V ensures that this is a profitable deviation.

To establish that p_{k+1} < 1, assume R accepts (z_k, s_{k+1}), the factions fight at s_{k+1}, and s_{k+1} = (1, 1). It suffices to show that z_k = 1 as this yields V_G(s_k) = βf_G. This, however, is a contradiction as G can always obtain at least f_G by fighting at s_k.

To see that z_k = 1, suppose z_k < 1. Since the factions fight at s_{k+1}, V_R(s_{k+1}) =
Proof: Lemma 4A establishes the result if the factions cases to consider. The proof of Lemma 3A establishes that 
Lemma 5A: If
\[ f_R + \beta(1 + \gamma)V. \]
PPC also holds at \( s_k \) since \( R \) accepts. Indeed, PPC must bind. If the PPC holds strictly, then \( R \) is sure to accept \((1, (1, 1 - \delta))\) for \( \delta \) small enough. This, however, yields a profitable deviation since
\[ V'_G(s_k) = 1 - z_k + \beta[f_G + \beta(1 + \gamma)V] = V_G(s_k) + \beta^2 \delta(1 + \gamma)V. \]
That PPC binds also implies
\[ z_k + \beta[1 - \varepsilon(1 - d_k)][f_R + \beta(1 + \gamma)V] = f_R + \beta d_k p_k (1 + \gamma)V + \beta(1 - d_k)(1 - \varepsilon)V_R(s_k). \]
The expression on the right is independent of \( G \)'s offer as \( R \) expects play to conform to \( E \) after any deviation. This and \( z_k < 1 \) mean that there exists a \( p' < 1 \) such that PPC binds at \((1, (1, p'))\). It follows that \( R \) is sure to accept \((1, (1, p' + \delta))\). If \( G \) fights at \((1, p' + \delta)\) (say by offering \((0, E)\)), then this deviation yields
\[ V'_G(s_k) = \beta[f_G + \beta(1 - p' - \delta)(1 + \gamma)V]. \] Consequently, \( V'_G(s_k) = V_G(s_k) = \beta^2(1 - p')(1 + \gamma)V - 1 + z_k - \delta(1 + \gamma)V. \)
Because PPC binds at \((z_k, (1, 1))\) and \((1, (1, p'))\), we have
\[ z_k + \beta[1 - \varepsilon(1 - d_k)][f_R + \beta(1 + \gamma)V] = 1 + \beta[1 - \varepsilon(1 - d_k)][f_R + \beta(1 - p')(1 + \gamma)V]. \] Algebra then gives
\[ \beta^2(1 - p')(1 + \gamma)V - 1 + z_k = \beta^2 \varepsilon (1 - d_k)(1 - p')(1 + \gamma)V > 0. \]
The strict inequality ensures \( V'_G(s_k) - V_G(s_k) > 0 \) for \( \delta \) small enough and hence that \((1, (1, p' + \delta))\) is a profitable deviation. This contradiction guarantees \( p_k = 1. \)

Lemma 5A shows that if \( R \) accepts \((z_k, s_{k+1})\), then there is a stage \( s' = (1, p') \) with \( p' < 1 \) which is payoff equivalent for \( R \) to \( s_{k+1} \), i.e., \( V_R(s_{k+1}) = V_R(s') \).

**Lemma 5A:** If \( d_k < 1 \) and \( R \) accepts \((z_k, s_{k+1})\), there exists a \( p' \in [0, 1) \) such that
\[ V_R(s_{k+1}) = f_R + \beta p'(1 + \gamma)V. \]

**Proof:** Lemma 4A establishes the result if the factions fight at \( s_{k+1} \). Observe further that there is nothing to show if
\[ V_R(s_{k+1}) < f_R + \beta(1 + \gamma)V \] since \( V_R(s_{k+1}) \geq f_R \). Assume, therefore, that the factions do not fight at \( s_{k+1} \) and that \( V_R(s_{k+1}) \geq f_R + \beta(1 + \gamma)V \). The proof of Lemma 3A establishes that
\[ 1 + \beta[f_R + \beta(1 + \gamma)V] > V_R(s_k). \]
There are now two cases to consider.

Case (i): \( V_R(s_k) \geq 1 + \beta f_R \). With \( 1 + \beta[f_R + \beta(1 + \gamma)V] > V_R(s_k) \), it suffices to show that \( z_k = 1 \) since \( V_R(s_{k+1}) = [V_R(s_k) - z_k]/\beta \). The first step is to demonstrate that the factions must fight in the continuation game. Arguing by contradiction, assume that the continuation games is peaceful. Then \( G \) has a profitable deviation.

To construct a profitable deviation, note that since
\[ 1 + \beta[f_R + \beta(1 + \gamma)V] > V_R(s_k) \]
$V_R(s_k) \geq 1 + \beta f_R$, there exists a $p' \in (0, 1)$ such that $V_R(s_k) = 1 + \beta[f_R + \beta p'(1 + \gamma)V]$. Assume $p' > 0$. Then $R$’s weak preference for accepting $(z_k, s_{k+1})$ over fighting at $s_k$ ensures that $R$ strictly prefers accepting $(1, (1, p' - \delta))$ at $s_k$ to fighting for a $\delta > 0$ sufficiently small. That is, $1 + \beta[f_R + \beta p'(1 + \gamma)V] = V_R(s_k) \geq f_R + \beta d_k p_k(1 + \gamma)V +\beta(1 - d_k)[(1 - \varepsilon)V_R(s_k) + \varepsilon V_R(s_{k+1})] > f_R + \beta d_k p_k(1 + \gamma)V + \beta(1 - d_k)[(1 - \varepsilon)V_R(s_k) + \varepsilon [f_R + \beta p'(1 + \gamma)V] where the strict inequality follows from $V_R(s_{k+1}) \geq f_R + \beta(1 + \gamma)V$ and $p' < 1$.

To ease the notation, define $s' = (1, p')$. Assume $p' > 0$ and recall that $q(s')$ is the smallest integer $m$ such that $(1 - \beta^m)V + \beta^m f_R \geq \Pi_R(s') = f_R + \beta p'(1 + \gamma)V$. Take $\delta$ sufficiently small so that $f_R + \beta(p' - \delta)(1 + \gamma)V \neq V - \beta^n(V - f_R)$ for any integer $n$. Lemma 2A ensures that $G$ can obtain at least $V'_G(s_k) \geq \beta[V - [f_R + \beta(p' - \delta)(1 + \gamma)V] + \beta^q(s')\gamma V]$ by offering $(1, (1, p' - \delta))$ and then eliminating $R$ as quickly as is peacefully possible at $(1, p' - \delta)$.

If, by contrast, $G$ plays according to its equilibrium strategy, it will take at least $q(s') + 1$ offers to eliminate $R$ since and $V_R(s_{k+1}) \geq f_R + \beta(1 + \gamma)V$. As a result, $V_G(s_k) \leq V + \beta^{q(s') + 1}\gamma V - V_R(s_k) = \beta[V + \beta^{q(s') + 1}\gamma V - [f_R + \beta p'(1 + \gamma)V]]$. But this means that $G$ has a profitable deviation since $V'_G(s_k) - V_G(s_k) \geq \beta^2(1 + \gamma)V + \beta^{q(s') + 1}(1 - \beta)\gamma V > 0$. This contradiction means that $p'$ must be zero if the continuation game is peaceful.

Suppose $p' = 0$. Then $V_R(s_k) = 1 + \beta f_R$. $R$’s acceptance now gives $1 + \beta f_R = V_R(s_k) \geq f_R + \beta d_k p_k(1 + \gamma)V + \beta[(1 - \varepsilon)V_R(s_k) + \varepsilon V_R(s_{k+1})]$. Since $V_R(s_{k+1}) \geq f_R + \beta(1 + \gamma)V > f_R$, we have $1 + \beta f_R = V_R(s_k) \geq f_R + \beta d_k p_k(1 + \gamma)V + \beta[(1 - \varepsilon)V_R(s_k) + \varepsilon f_R]$. Hence, $R$ strictly prefers accepting $(1 - \delta, E)$ at $s_k$ for $\delta$ sufficiently small. This leaves $G$ with $V'_G(s_k) = \delta + \beta[(1 + \gamma)V - f_R]$. That $V_R(s_{k+1}) > f_R$ also implies $s_{k+1} \neq E$ and that it therefore takes at least two offers to eliminate $R$. This means $V_G(s_k) \leq V - V_R(s_k) + \beta^2\gamma V$. As before, $G$ has a profitable deviation with $V'_G(s_k) - V_G(s_k) \geq \delta + \beta\gamma$. These profitable deviations mean that the continuation game cannot be peaceful.

Given that the factions must fight in the continuation game, we now argue by contradiction to show that $z_k = 1$. Assume the factions fight in the continuation game but $z_k < 1$. From Lemma 1A, $V_R(s_j) + V_G(s_j) \leq \max\{V; f_R + f_G + \beta(1 + \gamma)V\}$ for any
$s_j \neq E$ when the factions fight at $s_j$ or in the continuation game. Assume first that $V < f_R + f_G + \beta(1 + \gamma)V$. Then $V_G(s_{k+1}) > f_G$ if $s_{k+1} \neq (1, 1)$ since $G$ can always fight at $s_{k+1}$. This yields the contradiction $V_R(s_{k+1}) < f_R + \beta(1 + \gamma)V$. If $s_{k+1} = (1, 1)$, then $V_R(s_{k+1}) = 1 + \beta(1 + \gamma)V$ by Lemma 2A. But $V_G(s_{k+1}) + V_R(s_{k+1}) \leq f_G + f_R + \beta(1 + \gamma)V$, so $V_G(s_k) \leq f_G + f_R - 1 < 0$. This contradiction means that $V \geq f_G + f_R + \beta(1 + \gamma)V$ if $z_k < 1$.

Now assume $V \geq f_R + f_G + \beta(1 + \gamma)V$ with $z_k < 1$. If $p' > 0$, $G$ has a profitable deviation. As shown above, $R$ strictly prefers $\hat{s} \equiv (1, (1, \hat{p}))$ to fighting for $\hat{p} \equiv p' - \delta$ and $\delta$ small enough. As long as as we also choose $\delta$ so that $(1 - \beta^n)V + \beta^n f_R \neq f_R + \beta \hat{p}(1 + \gamma)V$ for any positive integer $n$, Lemma 2A ensures that $G$’s payoff at $(1, \hat{p})$ satisfies $V_G(\hat{s}) = \max\{\Pi_G(\hat{s}), F_G(\hat{s})\}$. Moreover, $\Pi_G(\hat{s}) > V - [f_R + \beta \hat{p}(1 + \gamma)V] \geq F_G(\hat{s})$. The weak inequality follows from the assumption that $V \geq f_R + f_G + \beta(1 + \gamma)V$ and the fact that $F_G(\hat{s}) + F_R(\hat{s}) = f_G + f_R + \beta(1 + \gamma)V$. Consequently, $G$’s payoff to offering $(1, \hat{s})$ and then eliminating $R$ as quickly as possible is $V_G(s_k) = \beta \Pi_G(\hat{s}) > \beta [V - (f_R + \beta \hat{p}(1 + \gamma)V)] = \beta [V - (f_R + \beta p'(1 + \gamma)V)] + \beta^2 \delta(1 + \gamma)V = V - V_R(s_k) + \beta^2 \delta(1 + \gamma)V$. But $V_R(s_k) + V_G(s_k) \leq V$. As a result, $V_G(s_k) > V_G(s_k)$ and proposing $(1, \hat{s})$ is a profitable deviation for $\delta$ sufficiently small. This contradiction implies that $z_k = 1$ if the factions fight in the continuation game and $p' > 0$.

If the factions fight and $p' = 0$, we have another contradiction. Once more, $V_R(s_k) = 1 + \beta f_R$ and $R$ strictly prefers $(1 - \delta, E)$ to fighting. Thus $V_G(s_k) \geq \delta + \beta'(1 + \gamma)V - f_R] = \delta + V - V_R(s_k) + \beta \gamma V$ and, consequently, $V_G(s_k) + V_R(s_k) > V$. But this is a contradiction as $V_G(s_k) + V_R(s_k) \leq 1 + \beta \max\{V, f_G + f_R + \beta(1 + \gamma)V\} = 1 + \beta V = V$. This contradiction implies $z_k = 1$. Using this in $z_k + \beta V_R(s_{k+1}) = V_R(s_k) < 1 + \beta [f_R + \beta(1 + \gamma)V]$ yields $V_R(s_{k+1}) < f_R + \beta(1 + \gamma)V$.

Case (ii): $V_R(s_k) < 1 + \beta f_R$. Clearly, $G$ can do no better than buying $R$ off in a single offer. Hence, $s_{k+1} = E$ and $V_R(s_{k+1}) = f_R$.□

The preceding lemmas characterize equilibrium play when $G$ lacks coercive power ($d_k = 1$); show that if $R$ accepts $(z_k, s_{k+1})$ at $s_k$, then $G$ can move play to an $\hat{s} = (1, \hat{p})$ with $\hat{p} < 1$; and that there exists an $\hat{s} = (1, \hat{p})$ with $\hat{p} < 1$ such that $V_R(s_{k+1}) = V_R(\hat{s}) = 43$.
Proof of Proposition 1: There is nothing to show if \( d_0 = 1 \) as Proposition 1 is simply a restatement of Lemma 2A when \( d_0 = 1 \). Assume then that \( d_0 < 1 \) and define \( s' = (1, p') \) and \( s'(\delta) = (1, p' + \delta) \) where \( V_R(s_1) = V_R(s') = f_R + \beta p'(1 + \gamma)V \). Lemma 5A guarantees that \((1, p')\) exists with \( p' \in [0, 1) \).

Let \( \mathcal{E} = \{\zeta, \sigma, \alpha\} \) be a pure-strategy MPE. Then PPC binds at \( s_0 \) if \( R \) accepts \( G \)'s equilibrium off \((z_0, s_1)\) at \( s_0 \) in \( \mathcal{E} \). Arguing by contradiction, assume that \( R \) strictly prefers \((z_0, s_1)\). This implies \( z_0 + \beta V_R(s_1) > f_R + \beta d_0 p_0 (1 + \gamma)V + \beta (1 - d_0)[(1 - \varepsilon) V_R(s_0) + \varepsilon V_R(s_1)] \). This is equivalent to \( z_0 + \beta[f_R + \beta p'(1 + \gamma)V] > f_R + \beta d_0 p_0 (1 + \gamma)V + \beta (1 - d_0)[(1 - \varepsilon) V_R(s_0) + \varepsilon[f_R + \beta p'(1 + \gamma)V]] \). If \( z_0 > 0 \), then \((z_0 - \delta, s_1)\) is an obviously profitable deviation. If \( z_0 = 0 \) but \( p' > 0 \), there exists a \( \delta > 0 \) such that \( R \) strictly prefers \((0, (1, p' - \delta))\), and this clearly would also be a profitable deviation for \( G \). If, however, \( z_0 = p' = 0 \), then \( V_R(s_0) = \beta f_R \). This is a contradiction as \( R \) can always obtain at least \( f_R \) by fighting. Thus, PPC binds at \( s_0 \) if \( R \) accepts \((z_0, s_1)\).

It is also the case that if the factions fight, they do so at \( s_0 \) or \( s_1 \). Suppose to the contrary that they fight at \( s_m \) where \( m > 1 \). Lemma 4A implies \( d_m = 1 \) and \( p_m < 1 \). Consequently, \( V_G(s_0) + V_R(s_0) = (1 - \beta^m)V + \beta^m[f_R + f_G + \beta(1 + \gamma)V - V] \). We show that \( G \) has a profitable deviation.

To construct this deviation, observe that since PPC binds at \((z_0, s_1)\) and \( V_R(s_1) = V_R(s') \), PPC also binds at \((z_0, s')\). That is, \( z_0 + \beta[f_R + \beta p'(1 + \gamma)V] = z_0 + \beta V_R(s_1) = f_R + \beta d_0 p_0 (1 + \gamma)V + \beta (1 - d_0)[(1 - \varepsilon) V_R(s_0) + \varepsilon V_R(s_1)] = f_R + \beta d_0 p_0 (1 + \gamma)V + \beta (1 - d_0)[(1 - \varepsilon) V_R(s_0) + \varepsilon[f_R + \beta p'(1 + \gamma)V]]. \) \( R \), therefore, strictly prefers \((z_0, s'(\delta))\) for \( \delta > 0 \).

If \( G \) offers \((z_0, s'(\delta))\) and then fights at \( s'(\delta) \), \( V'_{G}(s_0) + V'R(s_0) = 1 + \beta[f_R + f_G + \beta(1 + \gamma)V] \). Algebra then gives \( V'_R(s_0) - V_R(s_0) = V_R(s_0) - V'_R(s_0) + (\beta - \beta^m)[f_R + f_G - 1 + \beta \gamma V]. \) Furthermore, \( V'_R(s_0) = z_0 + \beta[f_R + \beta(p' + \delta)(1 + \gamma)V] = V_R(s_0) + \beta^2 \delta(1 + \gamma)V \). Hence, \( V'_R(s_0) - V_R(s_0) = (\beta - \beta^m)[f_R + f_G - 1 + \beta \gamma V] - \beta^2 \delta(1 + \gamma)V \). Assuming the contingent spoils are sufficiently large that \( f_R + f_G - 1 + \beta \gamma V > 0 \), then \( V'_R(s_0) - V_R(s_0) > 0 \) and
we have a profitable deviation for $\delta$ sufficiently small.

The fact that $G$ fights at $s_m$ ensures that the contingent spoils are this large. To establish $f_R + f_G - 1 + \beta\gamma V > 0$, consider first the case in which $f_R + \beta p_m(1 + \gamma)V \neq (1 - \beta^n)V - \beta^n f_R$ for any $n \geq 1$. Then Lemma 2A and the fact that $G$ fights at $s_m$ implies $f_G + \beta(1 - p_m)(1 + \gamma)V \geq \Pi_G(s_m) = V + \beta q(s_m)\gamma V - [f_R + \beta p_m(1 + \gamma)V]$. This leaves $f_R + f_G - 1 + \beta\gamma V \geq \beta q(s_m)\gamma V > 0$.

Now suppose $f_R + \beta p_m(1 + \gamma)V = V - \beta^n(V - f_R)$ for some $n \geq 1$ and let $\tilde{p}_{m+1}$ satisfy $f_R + \beta p_m(1 + \gamma)V = 1 + \beta[f_R + \beta\tilde{p}_{m+1}(1 + \gamma)V]$. $R$ is indifferent between fighting at $s_m$ and accepting $(1, \tilde{p}_{m+1})$ and, consequently, is sure to accept $(1, \tilde{p}_{m+1} + \delta)$ for any $\delta > 0$. This means that $G$ can obtain $V + \beta q(1, \tilde{p} + 1\gamma V - [f_R + \beta p_m(1 + \gamma)V] - \beta^2\delta(1 + \gamma)V$ by initially offering $(1, \tilde{p}_{m+1} + \delta)$ and then eliminating $R$ as quickly as is peacefully possible. Since $G$ at least weakly prefers to fight at $s_m$, this payoff must be no larger than what $G$ gets by fighting. That is, $f_G + \beta(1 - p_m)(1 + \gamma)V \geq V + \beta q(1, \tilde{p} + 1\gamma V - [f_R + \beta p_m(1 + \gamma)V] - \beta^2\delta(1 + \gamma)V$. Taking $\delta$ sufficiently small shows that $f_R + f_G - 1 + \beta\gamma V > 0$. Thus, the factions either fight at $s_0$ or $s_1$ or the equilibrium path is peaceful.

Now define a possibly negative $\hat{p}$ so that PPC binds if $G$ proposes $(1, \hat{s})$ at $s_0$ where $\hat{s} \equiv (1, \hat{p})$. That is, $\hat{p}$ satisfies $1 + \beta[f_G + \beta\hat{p}(1 + \gamma)V = f_R + \beta d_0p_0(1 + \gamma)V + \beta(1 - d_0)][(1 - \varepsilon)V_R(s_0) + \varepsilon[f_R + \beta(p(1 + \gamma)V]]$.

Turning directly to claim (ii), assume $B(s_0) > 1 + \beta f_R$. This condition ensures $B(s_0) > F_R(s_0)$ and, consequently, $\Pi_R(s_0) = B(s_0)$. We show first that if $R$ accepts $(z_0, s_1)$, then $z_0 = 1$, $V_R(s_0) = B(s_0)$, and $V_R(s_1) = f_R + \beta\hat{p}(1 + \gamma)V$. Assume $R$ accepts $(z_0, s_1)$. As shown above, $R$’s acceptance implies that PPC binds, so $z_0 + \beta[f_R + \beta p'(1 + \gamma)V = f_R + \beta d_0p_0(1 + \gamma)V + \beta(1 - d_0)][(1 - \varepsilon)V_R(s_0) + \varepsilon[f_R + \beta p'(1 + \gamma)V]$ where, recall, $V_R(s_1) = f_R + \beta p'(1 + \gamma)V$ for a $p' \in [0, 1]$.  

Since $R$ accepts $(z_0, s_1)$, the factions must fight at $s_1$ or the continuation game must be peaceful. Suppose the factions fight at $s_1$. Lemma 4A ensures $V_G(s_0) + V_R(s_0) = 1 + \beta[f_G + f_R + \beta(1 + \gamma)V]$. Arguing by contradiction to establish that $z_0 = 1$, assume the contrary. Because $z_0 < 1$ and PPC binds at $(z_0, s')$ and at $(1, \hat{s})$, we have $\hat{p} < p' < 1$. Assume further that $\hat{p} \geq 0$. By construction, $R$ is sure to accept $(1, \hat{s}(\delta))$ where $\hat{s}(\delta) \equiv (1, \hat{p} + \delta)$.
and $\delta > 0$.

It follows that proposing $(1, \hat{s}(\delta))$ and then fighting at $\hat{s}(\delta)$ is a profitable deviation for $G$. To verify this, observe that $V'_{G}(s_0) + V'_{R}(s_0) = 1 + \beta[f_G + f_R + \beta(1 + \gamma)V] = V_G(s_0) + V_R(s_0)$, so the deviation will be profitable if $V'_R(s_0) < V_R(s_0)$. Since PPC binds at $(z_0, s')$ and $(1, \tilde{s})$, it follows that $1 + \beta[1 - \varepsilon(1 - d_0)][f_R + \beta\bar{p}(1 + \gamma)V] = z_0 + \beta[1 - \varepsilon(1 - d_0)][f_R + \beta p'(1 + \gamma)V]$. This implies $V'_R(s_0) = 1 + \beta[f_R + \beta(\bar{p} + \delta)(1 + \gamma)V] < z_0 + \beta[f_R + \beta p'(1 + \gamma)V] = V_R(s_0)$ for $\delta$ sufficiently small. Hence $G$ has a profitable deviation if $z_0 < 1$ and $\hat{\rho} \geq 0$.

Suppose then that $z_0 < 1$ and $\hat{\rho} < 0$. Define $\tilde{z}$ so that PPC binds if $G$ offers $(\tilde{z}, E)$ at $s_0$. Then $R$ is sure to accept $(\tilde{z} + \delta, E)$ at $s_0$ where $\tilde{z} \leq z_0 < 1$ ensures $\tilde{z} + \delta$ is feasible for a sufficiently small $\delta$. If $G$ deviates in this way, then $V'_G(s_0) + V'_R(s_0) = V + \beta\gamma V > 1 + \beta[f_G + f_R + \beta(1 + \gamma)V] = V_G(s_0) + V_R(s_0)$. Thus $V'_G(s_0) - V_G(s_0) > V_R(s_0) - V'_R(s_0)$. To see that the latter difference is positive, use fact that PPC binds at $(\tilde{z}, E)$ and $(z_0, s')$ to obtain $\tilde{z} - z_0 - \bar{\sigma}^2 p'[1 - \varepsilon(1 - d_0)](1 + \gamma)V = 0$. This leaves $V'_R(s_0) = \tilde{z} + \delta + \beta f_R < z_0 + \beta[f_R + \beta p'(1 + \gamma)V] = V_R(s_0)$ for $\delta$ small enough. Thus, $G$ has a profitable deviation if $z_0 < 1$ and the factions fight at $s_1$.

It follows that $z_0 = 1$ and, consequently, $V_R(s_0) = 1 + \beta V_R(s_1)$ when the factions fight at $s_1$. Using the latter and PPC to solve for $V_R(s_0)$ gives $V_R(s_0) = B(s_0)$ and $V_R(s_1) = (B(s_0) - 1)/\beta = f_R + \beta\bar{p}(1 + \gamma)V$.

Suppose that $z_0 < 1$ and the continuation game is peaceful. Then $s_m = E$ for some $m$ (where $m = \infty$ if $R$ never agrees to $E$) and $V_G(s_0) + V_R(s_0) = V + \beta^m \gamma V$. We also have $m \geq 2$ since $V_R(s_0) > 1 + \beta f_R$. Once again $G$ has a profitable deviation. Suppose $\hat{\rho} \geq 0$ and that $G$ proposes $(1, \tilde{s}(\delta))$ and then eliminates $R$ as fast as peacefully possible. Since PPC binds at $(z_0, s')$ and $(1, \tilde{s})$, $1 - z_0 + \beta^2[1 - \varepsilon(1 - d_0)](\bar{p} - p')(1 + \gamma)V = 0$. This implies $\hat{\rho} < p'$ and consequently $q(s') \geq q(\tilde{s}(\delta))$ for $\delta$ sufficiently small. We also have $m \geq q(s') + 1$, so $V'_G(s_0) + V'_R(s_0) = V + \beta^{q(s')} + 1 \gamma V \geq V_G(s_0) + V_R(s_0)$. Moreover, $V'_R(s_0) = 1 + \beta[f_R + \beta(\bar{p} + \delta)(1 + \gamma)V] < z_0 + \beta[f_R + \beta p'(1 + \gamma)V] = V_R(s_0)$ for $\delta$ small enough. This leaves the contradiction $V'_G(s) > V_G(s_0)$.

Suppose that $z_0 < 1$ and $\hat{\rho} < 0$. Then $R$ is sure to accept $(1, E)$. If $G$ deviates in this
way, \( V'_G(s_0) + V'_R(s_0) = V + \beta \gamma V > V_G(s_0) + V_R(s_0) \). Moreover, \( V'_R(s_0) = 1 + \beta f_R < B(s_0) \leq V_R(s_0) \). Hence, this is a profitable deviation, and this contradiction ensures \( z_0 = 1 \) if the continuation game is peaceful. Repeating the argument above when \( z_0 = 1 \) gives \( V_R(s_0) = B(s_0) \) and \( V_R(s_1) = f_R + \beta \tilde{p}(1 + \gamma)V \) when the continuation game is peaceful.

In sum, if \( R \) accepts \( (z_0, s_1) \) and \( B(s_0) > 1 + \beta f_R \), then \( z_0 = 1 \), \( V_R(s_0) = B(s_0) = \Pi_R(s_0) \), and \( V_R(s_1) = f_R + \beta \tilde{p}(1 + \gamma)V \). It follows that \( \tilde{p} = \tilde{p} \) since \( V_R(s_0) = B(s_0) = 1 + \beta [f_R + \beta \tilde{p}(1 + \gamma)] \).

To show that the factions fight at \( s_0 \) in \( E \) if \( F_G(s_0) > \max\{\Pi_G(s_0), \beta F_G(s')\} \) argue by contradiction by assuming that the factions do not fight at \( s_0 \). Then \( z_0 = 1 \) and \( V_R(s_0) = B(s_0) \) since \( R \) accepts \( (z_0, s_1) \). If the factions fight at \( s_1, d_1 = 1 \) by Lemma 4A and \( V_R(s_1) = f_R + \beta p_1(1 + \gamma)V \). But \( V_R(s_1) = f_R + \beta \tilde{p}(1 + \gamma)V \), so \( s_1 = \tilde{s} \). It follows that \( G \) can profitably deviate by fighting at \( s_0 \) since \( F_G(s_0) > \beta F_G(\tilde{s}) \). This contradiction ensures that the factions do not fight at \( s_1 \).

If the continuation game is peaceful, \( V_G(s_0) \leq \Pi_G(s_0) \). Because \( F_G(s_0) > \Pi_G(s_0) \), \( G \) can profitably deviate by fighting at \( s_0 \). This contradiction means that the factions fight at \( s_0 \) with \( V_G(s_0) = F_G(s_0) \) and \( V_R(s_0) = F_R(s_0) \) when \( F_G(s_0) > \max\{\Pi_G(s_0), \beta F_G(\tilde{s})\} \).

Arguing again by contradiction to demonstrate that the factions fight at \( s_1 \) when \( \beta F_G(\tilde{s}) > \max\{\Pi_G(s_0), F_G(s_0)\} \), assume they do not. If the continuation game is peaceful, \( R \)'s acceptance of \( (z_0, s_1) \) means \( z_0 = 1 \) and \( V_R(s_0) = \Pi_R(s_0) \). This in turn leaves \( V_G(s_0) \leq V - B(s_0) + \beta^0(s_0) \gamma V = \Pi_G(s_0) \). Since \( z_0 = 1 \) and \( V_R(s_1) = f_R + \beta \tilde{p}(1 + \gamma)V \), \( R \) is sure to accept \( (1, 1, \tilde{p} + \delta) \) for any \( \delta > 0 \). \( G \), therefore, can obtain \( V'_G(s_0) = \beta F_G(\tilde{s}) - \delta \beta^2(1 + \gamma)V \) by offering \( (1, 1, \tilde{p} + \delta) \) and then fighting. This is clearly profitable for \( \delta \) is sufficiently small.

If the factions fight at \( s_0 \), \( V_R(s_0) = F_R(s_0) = F_R(s_0) \). Recalling that \( B(s_0) > F_R(s_0) \) in claim (ii), algebra shows that \( R \) strictly prefers \( (1, \tilde{s}) \) to fighting. As a result, \( G \) can deviate by proposing \( (1, \tilde{s}) \) and then fight there. This brings \( V'_G(s_0) = \beta F_G(\tilde{s}) \) and is clearly profitable. This contradiction ensures that the factions fight at \( s_1 \) when \( \beta F_G(\tilde{s}) > \max\{\Pi_G(s_0), F_G(s_0)\} \).

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The factions’ payoffs follow immediately. Fighting at $s_1$ means that $R$ accepts $(z_0, s_1)$. As a result, $z_0 = 0$, $s_1 = \tilde{s}$, $V_C(s_0) = 1 - z_0 + \beta F_G(s_1) = \beta F_G(\tilde{s})$, and $V_R(s_0) = B(s_0)$.

Finally, assume $\Pi_G(s_0) > \max\{F_G(s_0), \beta F_G(\tilde{s})\}$ and that the factions fight at $s_0$ or $s_1$. The former means $V_G(s_0) = F_G(s_0)$. As just shown, this implies $V_R(s_0) = F_R(s_0) < B(s_0)$ and that $R$ is sure to accept $(1, \tilde{s})$. It follows that $G$ can deviate to $(1, \tilde{s})$ and then eliminate $R$ as quickly as possible. This yields $V_G'(s_0) = \Pi_G(s_0)$ and would be a profitable deviation.

If the factions fight at $s_1$, then, as before, $z_0 = 1$, $s_1 = \tilde{s}$, $V_R(s_0) = B(s_0)$, and $V_G(s_0) = \beta F_G(\tilde{s})$. If, however, $G$ eliminates $R$ as quickly as possible at $\tilde{s}$, it obtains $V_G'(s_0) = V - B(s_0) + \beta^{q(\tilde{s})+1}\gamma V = \Pi_G(s_0)$. $G$ again has a profitable deviation which means that the equilibrium path is peaceful when $\Pi_G(s_0) > \max\{F_G(s_0), F_G(s_0)\}$.

As for $G$’s payoff, the fact that the continuation game is peaceful ensures $V_G(s_0) \leq \Pi_G(s_0)$. We also have $z_0 = 1$ and $s_1 = \tilde{s}$. By assumption $B(s_0) \neq V - \beta^n(V - f_R)$ for any integer $n \geq 1$. $G$, therefore, can eliminate $R$ from $(1, \tilde{p} + \delta)$ in the same number of rounds as from $\tilde{s}$ if $\delta$ is sufficiently small, i.e., $q(\tilde{s}) = q(\tilde{s}(\delta))$ where $\tilde{s}(\delta) = (1, \tilde{p} + \delta)$. Proposing $\tilde{s}(\delta)$ and then eliminating $R$ as quickly as possible cannot be profitable, so $V_G(s_0) \geq V - B(s_0) + \beta^{q(s_0)}\gamma V - \delta \beta^2(1 + \gamma)V = \Pi_G(s_0) - \delta \beta^2(1 + \gamma)V$ for arbitrarily small $\delta$. Hence, $V_G(s_0) = \Pi_G(s_0)$, and this establishes claim (ii).

Now consider claim (i) with $B(s_0) < 1 + \beta f_R$. This inequality implies $F_R(s_0) > B(s_0)$. It follows that $R$ accepts $(z_0, s_1)$. To establish this, suppose the factions fight at $s_0$. Then $V_R(s_0) = F_R(s_0) = F_R(s_0)$. Since $F_G(s_0) + F_R(s_0) \leq f_G + f_R + \beta(1 + \gamma)V$, then $V_G(s_0) = F_G(s_0) \leq f_G + f_R + \beta(1 + \gamma)V - F_R(s_0)$. Moreover, $R$ strictly prefers $(F_R(s_0) - \beta f_R + \delta, E)$ to fighting for any $\delta > 0$ since $F_R(s_0) + \delta > f_R + \beta d_0 p_0 (1 + \gamma)V + \beta (1 - d_0)((1 - \varepsilon) F_R(s_0) + \varepsilon f_R) = F_R(s_0)$. Deviating in this way brings $G$ a payoff of $V_G'(s_0) = V + \beta \gamma V - \Pi_R(s_0) - \delta$. Hence $G$ has a profitable deviation if $\delta$ is sufficiently small. This contradiction means that the factions cannot fight at $s_0$.

Since $R$ accepts $(z_0, s_1)$, $V_R(s_1) = f_R + \beta p'(1 + \gamma)V$ for a $p' \in [0, 1)$ and PPC binds at $(z_0, s_1)$. This leaves $V_R(s_0) = z_0 + \beta[f_R + \beta p'(1 + \gamma)V] = f_R + \beta d_0 p_0 (1 + \gamma)V + \beta (1 - d_0)((1 - \varepsilon) V_R(s_0) + \varepsilon [f_R + \beta p'(1 + \gamma)V])$. 

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Arguing by contradiction to show that factions do not fight at $s_1$, suppose they do. Then $V_G(s_0) + V_R(s_0) = 1 + \beta[f_G + f_R + \beta(1 + \gamma)V]$, and $G$ has a profitable deviation. Assuming that $\hat{\rho} \geq 0$, $R$ is sure to accept $(1, \hat{s}(\delta))$ for any $\delta > 0$. Suppose then that $G$ offers this and fights at $(1, \hat{s}(\delta))$. We have $V_G'(s_0) + V_R'(s_0) = 1 + \beta[f_G + f_R + \beta(1 + \gamma)V]$, so this will be a profitable deviation for $G$ if $V_R'(s_0) < V_R(s_0)$.

Because PPC binds at $(1, \hat{s})$ and at $(z_0, s')$, $1 + \beta[1 - \varepsilon(1 - d_0)][f_G + \beta\hat{\rho}(1 + \gamma)V = z_0 + \beta(1 - \varepsilon(1 - d_0)][f_G + \beta\hat{\rho}(1 + \gamma)V].$ If $z_0 < 1$, then $1 - z_0 < \beta^2(p' - \hat{\rho} - \delta)(1 + \gamma)V$ for a $\delta$ small enough. This implies $V_R'(s_0) = 1 + \beta[f_G + \beta(\hat{\rho} + \delta)(1 + \gamma)V < z_0 + \beta[f_G + \beta\hat{\rho}(1 + \gamma)V] = V_R(s_0).$ Hence $G$ has a profitable deviation if $\hat{\rho} \geq 0$ and $z_0 < 1$.

Assume $z_0 = 1$. Using $V_R(s_0) = 1 + \beta V_R(s_1)$ and the fact that PPC binds, solving for $V_R(s_0)$ gives $V_R(s_0) = B(s_0) < F_R(s_0)$. $R$, therefore is sure to accept $(F_R(s_0) - \beta f_R, E)$ as $F_R(s_0) > B(s_0) = V_R(s_0) \geq f_R + \beta d_0 p_0 (1 + \gamma)V + \beta (1 - d_0)(1 - \varepsilon)V_2(s_0) + \varepsilon f_R$. Deviating in this way brings $V_G'(s_0) = 1 + \beta(1 + \gamma)V - F_R(s_0) > \beta(1 + \gamma)V - \beta f_R$ where the strict inequality follows from the condition defining the case. If, however, $G$ fights at $s_1$ with $z_0 = 1$, its payoff is bounded by $V_G(s_0) \leq \beta[f_G + \beta(1 + \gamma)V]$. Algebra shows that $\beta(1 + \gamma)V - \beta f_R > \beta[f_G + \beta(1 + \gamma)V]$. Hence, $G$ has a profitable deviation if the factions fight at $s_1$ and $\hat{\rho} \geq 0$.

Assume $\hat{\rho} < 0$. Then $R$ strictly prefers $(1, E)$. Deviating in this way brings $V_G'(s_0) = \beta(1 + \gamma)V - \beta f_R$ which, as just shown, is a profitable deviation. These contradictions imply that the factions cannot fight at $s_1$. The continuation game must therefore be peaceful.

If $s_1 = E$, we are done. To wit, $V_R(s_0) = z_0 + \beta V_R(E) = f_R + \beta d_0 p_0 (1 + \gamma)V + \beta (1 - d_0)(1 - \varepsilon)V_2(s_0) + \varepsilon V_R(E)].$ Hence, $V_R(s_0) = F_R(s_0)$ and $z_0 = F_R(s_0) - \beta f_R$. This in turn leaves $V_G(s_0) = \Pi_G(s_0)$.

To show that $s_1 = E$, assume the continuation game is peaceful but $s_1 \neq E$. Then $V_G(s_0) + V_R(s_0) = V + \beta^m \gamma V$ where $s_m = E$ for some $m \geq 2$. $G$ again has a profitable deviation. Suppose $\hat{\rho} \geq 0$, $z_0 < 1$, and $G$ deviates by proposing $(1, \hat{s}(\delta))$ and eliminating $R$ as quickly as is peaceably possible with $\delta$ chosen so that $f_R + \beta(\hat{\rho} + \delta)(1 + \gamma)V \neq V - \beta^n (V - f_R)$ for any integer $n \geq 1$. Since $z_0 < 1$, it follows that $\hat{\rho} \leq p'$ and as a
result \( q(s_1) = q(s') \geq q(\widehat{s}(\delta)) \) and \( m \geq q(\widehat{s}(\delta)) + 1 \) for \( \delta \) sufficiently small. This leaves
\[ V'_G(s_0) + V'_R(s_0) = V + \beta^q(\widehat{s}(\delta))\gamma V \geq V + \beta^m \gamma V = V_G(s_0) + V_R(s_0). \]
\( G \) therefore has a profitable deviation if \( V'_R(s_0) < V_R(s_0) \). Repeating the argument above shows this to be the case when \( z_0 < 1 \) and \( \widehat{p} \geq 0 \).

If \( z_0 = 1 \) and \( \widehat{p} \geq 0 \), repeating the argument above shows that \( R \) strictly prefers \((F_R(s_0) - \beta f_R, E)\) to fighting. Thus, \( V'_G(s_0) + V'_R(s_0) = V + \beta \gamma V \geq V + \beta^m \gamma V = V_G(s_0) + V_R(s_0) \). Furthermore, \( V'_R(s_0) = \Pi(s_2(s_0)) < 1 + \beta f_R = V_R(s_0) \). Hence, \( V'_G(s_0) > V_G(s_0) \).

Finally, assume \( \widehat{p} < 0 \) and recall that PPC binds if \( G \) proposes \((\widehat{z}, E)\) at \( s_0 \). Then \( R \) is sure to accept \((\widehat{z} + \delta, E)\) for any \( \delta > 0 \), and \( V'_G(s_0) + V'_R(s_0) = V + \beta \gamma V \geq V + \beta^m \gamma V = V_G(s_0) + V_R(s_0) \). Repeating the arguments above also shows \( \widehat{z} - z_0 - \beta^2[1 - \epsilon(1 - d_0)]p'(1 + \gamma)V = 0 \). As a result, \( \widehat{z} + \delta + \beta f_R < z_0 + \beta[f_R + \beta p'(1 + \gamma)V] \). Hence, \( V'_G(s_0) > V_G(s_0) \) and \( G \) has a profitable deviation if \( s_1 \neq E_1 \). \( \Box \)

It is now straightforward to characterize \( G \)'s payoffs as a function of the contingent gain \( \gamma \) and establish Proposition 2:

**Proof of Proposition 2.** Proposition 1 ensures that \( G \) always prefers to eliminate \( R \) peacefully when \( B(s_0, \gamma) < 1 + \beta f_R \). Letting \( \widetilde{\gamma} \) to solve \( B(\gamma) = 1 + \beta f_R \) (where we suppress the stage in order to simplify the notation), \( \Pi_G(\gamma) > \max\{F_G(\gamma), \beta F_G(\widetilde{s}(\gamma))\} \) for \( \gamma < \widetilde{\gamma} \).

When \( B(\gamma) \geq 1 + \beta f_R \), we have \( \Pi_G(\gamma) = V + \beta^q(\gamma) V - B(\gamma) \) where recall \( q(\gamma) \) is the integer such that \( V - \beta^q(\gamma)(V - f_R) \geq B(\gamma) > V - \beta^q(\gamma)^{-1}(V - f_R) \). This implies \( \Pi_G(\gamma) \equiv (V - B(\gamma))[1 + \beta \gamma V/(V - f_R)] < \Pi_G(\gamma) \leq (V - B(\gamma))[1 + \gamma V/(V - f_R)] \equiv \Pi_G(\gamma) \). Both \( \Pi_G(\gamma) \) and \( \Pi_G(\gamma) \) are quadratic and concave in \( \gamma \) with \( \Pi_G(\gamma) > \max\{F_G(\gamma), \beta F_G(\widetilde{s}(\gamma))\} \). Moreover, \( F_G(\gamma) \) and \( \beta F_G(\widetilde{s}(\gamma)) \) are linear in \( \gamma \). Finally recall from the discussion preceding the statement of Proposition 2 that \( 0 = \Pi_G(\gamma) \equiv \max\{F_G(\gamma), \beta F_G(\overline{s}(\gamma))\} \). All of this ensures that there exists a unique \( \gamma > \gamma \) such that \( \Pi_G(\gamma) > \max\{F_G(\gamma), \beta F_G(\widetilde{s}(\gamma))\} \) for all \( \gamma < \gamma \) and a unique \( \gamma > \gamma \) such that \( \Pi_G(\gamma) > \max\{F_G(\gamma), \beta F_G(\widetilde{s}(\gamma))\} \) for all \( \gamma > \gamma \). \( \Box \)

**Proof of Proposition 4.** Note that \( F_R(s_k, \gamma) \) is strictly increasing in \( \gamma \). As a result, a unique (and possibly negative) \( \gamma \) satisfies \( F_R(s_k, \gamma) = 1 + \beta f_R \). Define \( \gamma = \max\{0, \gamma\} \). Then \( F_R(s_k, \gamma) < 1 + \beta f_R \) for \( \gamma \in (k, \gamma) \), and \( G \) eliminates \( R \) peacefully by part (i) of
Proposition 1.

Take $\gamma \geq \bar{\gamma}$. $F_G(s_k, \gamma)$ is linearly increasing in $\gamma$. As for $F_G(s_1, \gamma)$, solving $B(s_k) = 1 + \beta \pi_{k+1}(1 + \gamma)V$ for $\pi_{k+1}$ and substituting it into $F_G(\pi_{k+1})$ shows $F_G(\pi_{k+1})$ to be linear as well. Turning to $\Pi_G(s_k)$, recall that $\Pi_G(s_k) = (1 + \beta^q(s_k, \gamma))V - \Pi_R(s_k)$ when $\Pi_R(s_k) < V$. We can then use $(1 - \beta^q(s_k - 1))V + \beta^q(s_k - 1)f_R < \Pi_R(s_k) \leq (1 - \beta^q(s_k))V + \beta^q(s_k) f_R$ to bound $\Pi_G(s_k)$ by eliminating $\beta^q(s_k)$. This gives $\Pi_G(s_k) < \Pi(s_k) \leq \Pi_G(s_k)$ where

$$\Pi_G(s_k) = [(1 + \beta \gamma)V - f_R] \left[ \frac{V - \Pi_R(s_k)}{V - f_R} \right]$$

$$\Pi(s_k) = [(1 + \gamma)V - f_R] \left[ \frac{V - \Pi_R(s_k)}{V - f_R} \right].$$

The bounds $\Pi_G(s_k)$ and $\Pi(s_k)$ are quadratic and concave in $\gamma$. Moreover, $\Pi_G(s_k) > \max\{F_G(s_k), \beta F_G(\pi_{k+1})\}$ at $\bar{\gamma}$. We also have $\Pi_R(s_k) = V$ and $\Pi_G(s_k) = \Pi_G(s_k) = 0$ at $\gamma = \bar{\gamma}$ where $\gamma = \bar{\gamma}$ solves $\Pi_R(s_k, \bar{\gamma}) = V$. This leaves $\Pi_G(s_k, \bar{\gamma}) < \max\{F_G(s_k, \bar{\gamma}), \beta F_G(\pi_{k+1}, \bar{\gamma})\}$.

It follows that there exists $\gamma$ and $\bar{\gamma}$ such that $0 \leq \bar{\gamma} < \gamma < \bar{\gamma} < \bar{\gamma}$ and the factions fight when $\bar{\gamma} < \gamma$. When $0 < \gamma < \bar{\gamma}$, $G$ eliminates $R$ as fast as is peacefully possible.\]

SCRAPS relating to the proofs of the comparative statics:

At these points, the number of rounds it takes to buy $R$ off increases by one and $F_G(s_k) - \Pi_G(s_k)$ discontinuously jumps down to $F_G(s_k) - \Pi_G(s_k)$ where $\Pi_G(s_k) \equiv [(1 + \beta \gamma)V][V - \Pi_R(s_k)]/[V - f_R].$\[35\] The difference $F_G(s_k) - \Pi_G(s_k)$ is also is increasing in $d_k$, and, as a result, $F_G(s_k) - \Pi_G(s_k)$, is also increasing for large enough changes in $d_k$. Analogous results hold for $\beta F_G(\pi_{k+1}) - \Pi_G(s_k)$. Summarizing,

To be more precise, $F_G(s_k) + \Pi_R(s_k)$ in Eq (2) increasing in in $d_k$ as is the midwhen $p_k$ and $\varepsilon$ are small. Moreover, the number of rounds it takes to buy $R$ off is constant in a neighborhood of $d_k$ as long as $\Pi_R(s_k) \neq (1 - \beta^m)V + \beta f_R$ for any integer $m$. It follows that $F_G(s_k) - \Pi_G(s_k)$ is increasing in $d_k$ except at these point. At these points, the number of rounds it takes to buy $R$ off increases by one and $F_G(s_k) - \Pi_G(s_k)$ jumps down to

\[35\] Solving $\Pi_R(s_k) > (1 - \beta^q(s_k - 1))V + \beta^q(s_k - 1)f_R$ to bound $\beta^q(s_k)$ and then using this bound in the expression for $\Pi(s_k)$ gives $\Pi_G(s_k) > \Pi_G(s_k)$.
\[ F_G(s_k) - \Pi_G(s_k) \] where \( \Pi_G(s_k) \equiv [(1 + \beta\gamma)V][V - \Pi_R(s_k)]/[V - f_R]. \] The difference \( F_G(s_k) - \Pi_G(s_k) \) is also increasing in \( d_k \), and, as a result, \( F_G(s_k) - \Pi_G(s_k) \), is also increasing for large enough changes in \( d_k \). Analogous results hold for \( \beta F_G(s_{k+1}) - \Pi_G(s_k) \).

\[ \text{36 Solving } \Pi_R(s_k) > (1 - \beta^{\eta(s_k)-1})V + \beta^{\eta(s_k)-1}f_R \text{ to bound } \beta^{\eta(s_k)} \text{ and then using this bound in the expression for } \Pi_G(s_k) \text{ gives } \Pi_G(s_k) > \Pi_G(s_k). \]


